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MAGNETOHYDRODYNAMIC UNSTEADY FREE CONVECTION FLOW PAST AN INFINITE VERTICAL PLATE WITH CONSTANT SUCTION AND HEAT SINK

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The study of unsteady hydromagnetic free convective flow of viscous incompressible and electrically conducting fluids past an infinite vertical porous plate in the presence of constant suction and heat absorbing sinks has been made. Approximate solutions have been derived for the velocity and temperature fields, mean skin-friction and mean rate of heat transfer using multi-parameter perturbation technique. It is observed that increase in magnetic field strength decreases the velocity of the fluid. Also the mean skin-friction and mean rate of heat transfer of the conducting fluid decrease with increase in magnetic field strength.

Key Words : MHD; Thermal Energy; Unsteady; Porous Flat Plate; Sinks; Constant Suction

1. INTRODUCTION

The unsteady free convection flow past an infinite plate with constant suction and heat sources has been studied by Pop and Soundalgekar[1]. The effect of magnetic field on free convective flow of electrically conducting fluids past a semi-infinite flat plate has been analysed by Gupta[2], Singh and Cowling[3], Nanda and Mohanty[4]. Sacheti, Chandran and Singh[5] have obtained an exact solution for the unsteady MHD problem. MHD free convective flow with Hall current in a porous medium for electrolytic solution (viz. salt water) has been studied by Sattar and Alam[6]. But they have neither considered the effect of constant suction nor included the heat absorbing sink and viscous dissipation. The propagation of thermal energy through mercury and electrolytic solution in the presence of external magnetic field and heat absorbing sinks has wide range of applications in chemical and aeronautical engineering, atomic propulsion, space science etc. Our objective in the present paper is to study the heat transfer in mercury ($P_r = 0,.025$) and electrolytic solution ($P_r = 1.0$) past an infinite porous plate with constant suction in the presence of uniform transverse magnetic field and heat sink.

2. MATHEMATICAL ANALYSIS

Let the x'-axis be taken in the vertically upward direction along the infinite vertical plate and y'-axis normal to it. Neglecting the induced magnetic field and applying Boussinesq's approximation, the equations of the flow can be written as :

Continuity :

$$\frac{\partial V'}{\partial y'}=0 \quad \text{... (1)}$$

i.e. $$V'=V_0' \text{ (constant)} \quad \text{.... (2)}$$

Momentum :

$$\frac{\partial u'}{\partial t'}+V'\frac{\partial u'}{\partial y'}=g\,\beta\,(T'-T_\infty')+\frac{\nu\,\partial^2 u'}{\partial y'^2}-\frac{\sigma B_0^2}{\rho}u' \quad \text{... (3)}$$

Energy :

$$\frac{\partial T'}{\partial t'}+V'\frac{\partial T'}{\partial y'}=k\frac{\partial^2 T'}{\partial y'^2}+S'(T'-T_\infty')+\frac{\nu}{C_p}\left(\frac{\partial u'}{\partial y'}\right)^2, \quad \text{... (4)}$$

On disregarding the Joulean heat dissipation. The boundary conditions of the problem are:

$$u'=0,\; V'=-V_0',\; T'=T_\omega'+\varepsilon\,(T_\omega'-T_\infty')\,e^{i\,\omega' t'} \text{ at } y'=0$$

$$u'\to 0,\; T'\to T_\infty' \text{ as } y'\to\infty \quad \text{... (5)}$$

3. Method of Solution

Introducting the following non-dimensional variables and parameters,

$$y=\frac{y'V_0'}{\nu},\; t=\frac{t'V_0'^2}{4\,\nu},\; \omega=\frac{4\,\nu\,\omega'}{V_0'^2},\; u=\frac{u'}{V_0'},\; \nu=\frac{\eta_0}{\rho}$$

$$T=\left(\frac{T'-T_\infty'}{T_\omega'-T_\infty'}\right),\; P_r=\frac{\nu}{K},\; G=\frac{\nu\,g\,\beta\,(T_\omega'-T_\infty')}{V_0'^2} \quad \text{... (6)}$$

$$S=\frac{4S'\nu}{V_0'^2},\; E_c=\frac{V_0'^2}{C_p\,(T_\omega'-T_\infty')},\; M=\left(\frac{\sigma B_0^2}{\rho}\right)\frac{\nu}{V_0^2},\; K=\frac{K_0}{\rho\,C_p},$$

where *Pr, G, S,* E_c *and M are respectively the prandtl number, Grashof number, Sink strength, Eckert number and Hartmann number. With the help of eqs. (5) and (6), eqs. (2) and (3) become*

$$\frac{1}{4}\frac{\partial u}{\partial t} - \frac{\partial u}{\partial y} = GT + \frac{\partial^2 u}{\partial y^2} - Mu \quad \text{... (7)}$$

and

$$\frac{P_r}{4}\frac{\partial T}{\partial t} - P_r\frac{\partial T}{\partial y} = \frac{\partial^2 T}{\partial y^2} + \frac{P_r S}{4} T + P_r E_C \left(\frac{\partial u}{\partial y}\right)^2 \quad \text{... (8)}$$

and the modified boundary conditions are

$$u = 0,\ T = 1 + \varepsilon e^{i\omega t} \text{ at } y = 0 \quad \text{... (9)}$$

and

$$u \to 0, T \to 0 \text{ as } y \to \infty.$$

To solve eqs. (7) and (8), we assume ω to be very small and the velocity and temperature in the neighbourhood of the plate as

$$u(y, t) = u_0(y) + \varepsilon e^{i\omega t} u_1(y),$$

and

$$T(y, t) = T_0(y) = \varepsilon e^{i\omega t} T_1(y). \quad \text{... (10)}$$

Putting (10) in eqs. (7) and (8), equating harmonic and non-harmonic terms and neglecting the co-efficient of ω^2, we get

$$u_0'' + u_0' - Mu_0 = -GT_0,\ T_0'' + P_r T_0' + \frac{P_r S}{4} T_0 = -P_r E_e \left(\frac{\partial u_0}{\partial y}\right)^2 \quad \text{... (11)}$$

and

$$u_1'' + u_1' - \frac{i\omega}{4} u_1 - Mu_1 = -GT_1,$$

and

$$T_1'' + P_r T_1' - \frac{P_r}{4}(i\omega - S) T_1 = -2 P_r E_c \left(\frac{\partial u_0}{\partial y}\right)\left(\frac{\partial u_1}{\partial y}\right). \quad \text{... (12)}$$

Using multiparameter perturbation technique and assuming $\varepsilon_c << 1$, we write,

$$u_0 = u_{00} + E_c u_{01},\ T_0 = T_{00} + E_c T_{01} \quad \text{... (13)}$$

and

$$u_1 = u_{10} + E_c u_{11},\ T_1 = T_{10} + E_c T_{11} \quad \text{... (14)}$$

Using eqs. (13) and (14) in the eqs. (11) and (12) and equating the coefficients of E_c^0 and E_c^1 only, we get the following sets of differential equations for

$$u_{00}, u_{10}, T_{00}, T_{10} \text{ and } u_{01}, u_{11}, T_{01}, T_{11},$$

$$u_{00}'' + u_0' - Mu_{00} = -GT_{00},\quad u_{10}'' + u_{10}' - \frac{i\omega}{4} u_{10} - Mu_{10} = -GT_{10} \quad \text{... (15)}$$

$$T_{00}'' + P_r T_{00}' + P_r \frac{S}{4} T_{00} = 0, \quad T_{10}'' + P_r T_{10}' - \frac{P_r}{4}(i\,\omega - S)\,T_{10} = 0 \qquad \text{... (16)}$$

and $$u_{01}'' + u_{01}' - M u_{01} = -\,G T_{01}, \quad u_{11}'' + u_{11}' - \frac{i\,\omega}{4} u_{11} - M u_{11} = -\,G T_{11} \qquad \text{... (17)}$$

$$T_{01}'' + P_r T_{01} + \frac{P_r S}{4} T_{01} = -\,P_r (u_{00}')^2, \; T_{11}' + P_r T_{11}' - \frac{P_r}{4}(i\,\omega - S)\,T_{11}$$

$$= -\,2P_r \left(\frac{\partial u_0}{\partial y}\right)\left(\frac{\partial u_{10}}{\partial y}\right) \qquad \text{... (18)}$$

Solving these differential eqs. (15-18) with aid of the corresponding boundary conditions and then substituting the values in the relations (13) and (14), we obtain the mean velocity u_0 and mean temperature T_0 as well as $u, T,$ as

$$u_0 = \frac{4P_r E_c G^3}{(a_1^2 - a_1 - M)^2}\left[\frac{a_2^2 e^{-2a_2 y}}{B_1 (4a_2^2 - 2a_2 - M)} + \frac{a_1^2 e^{-2a_2 y}}{B_2 (4a_1^2 - 2a_1 - M)} - \frac{2a_1 a_2 e^{(-(a_1 + a_2) y}}{B_3 (a_1 + a_2)^2 - (a_1 + a_2) - M)}\right]$$

$$-\,\frac{G(1 + B_4 E_c G^2)\, e^{-a_1 y}}{a_1^2 - a_1 - M} + B_5 e^{-a_2 y}, \qquad \text{... (19)}$$

$$T_0 = (1 + B_4 E_c G^2)\, e^{-a_1 y} - \frac{4P_r E_c G^2}{(a_1^2 - a_1 - M)^2}$$

$$\left[\frac{a_2^2 e^{-2a_2 y}}{B_1} + \frac{a_1^2 e^{-2a_1 y}}{B_2} - \frac{2a_1 a_2 e^{-(a_1 + a_2) y}}{B_3}\right] \qquad \text{... (20)}$$

$$u_1 = D_6 e^{-a_4 y} - \frac{G e^{-a_3 y}}{a_3^2 - a_3 - \left(i \frac{\omega}{4} - M\right)} - E_c G$$

$$[D_7 e^{-a_3 y} - D_8 e^{-(a_2 + a_4) y} + D_9 e^{-(a_1 + a_4) y}$$

$$+ D_{10} e^{-(a_1 + a_2 + a_4) y} + D_{11} e^{-(a_2 + a_3) y} + D_{12} e^{-(a_1 + a_3) y} + D_{13} e^{-(a_1 + a_2 + a_3) y}\Big] \qquad \text{... (21)}$$

and $$T_1 = e^{-a_3 y} + E_c\Big[F D_1 e^{a_3 y} - D_2 e^{-(a_2 + a_4) y} + D_3 e^{-(a_1 + a_4) y}$$

$$+ B_6 e^{-(a_1 + a_2 + a_4) y} = D_4 e^{-(a_2 + a_3) y} + D_5 e^{-(a_1 + a_3) y} + B_7 e^{-(a_1 + a_2 + a_3) y} \Big] \quad \dots (22)$$

where $a_1 = \frac{1}{2}\left[P_r + \sqrt{P_r^2 - P_r S} \right], \; a_2 = \frac{1}{2} [1 + \sqrt{1 + 4M}\,],$

$$a_3 = \frac{1}{2}\left[P_r + \sqrt{(P_r^2 + P_r (i\,\omega - S))} \right], \; a_4 = \frac{1}{2} [1 + \sqrt{(1 + 4M) + i\,\omega}\,]$$

$B_1, B_2, B_3, B_4, B_5, B_6, D_1, D_2, \dots D_{13}$ are constants and not given to save space.

Separating real and imaginary parts of the velocity and temperature expressions (10) and taking only the real parts, we arrive at the velocity and temperature fields in terms of the fluctuating parts in the form.

$$u = u_0 + \varepsilon (M_r \cos \omega t - M_i \sin \omega t) \quad \dots (23)$$

$$T = T_0 + \varepsilon (T_r \cos \omega t - T_i \sin \omega t) \quad \dots (24)$$

The transient velocity and temperature for $\omega t = \pi/2$ are given by

$$u = u_0 - \varepsilon M_1 \text{ and } T = T_0 - \varepsilon T_i \quad \dots (25)$$

Skin-Friction and Rate of Heat Transfer

The skin friction at the plate in dimensionless form is given by

$$\tau_\omega = \left(\frac{\partial u}{\partial y} \right)_{y=0} = u_0' (0) + \omega e^{i \omega t} u_1' (0). \quad \dots (26)$$

Separating real and imaginary parts of eq. (17) and taking real part only, we have

$$\tau_\omega = \tau_\omega^m + \varepsilon \, | B | \cos (\omega t + \alpha), \quad \dots (27)$$

where $| B | = \sqrt{B_r^2 + B_i^2}, \; \alpha = \tan^{-1} \left(\frac{B_i}{B_r} \right).$ ... (28)

Similarly, the rate of heat transfer at the plate is given by

$$q_\omega = \left(\frac{\partial T}{\partial y} \right)_{y=0} = T_0' (0) + \varepsilon e^{i \omega t} T_1' (0) \quad \dots (29)$$

and on further simplification we have

$$q_\omega = q_\varepsilon^m + \omega \, | H | \cos (\omega t + \gamma) \quad \dots (30)$$

where $| H | = \sqrt{H_r^2 + H_i^2}, \; \gamma = \tan^{-1} \left(\frac{H_i}{H_r} \right).$ ... (31)

4. Results and Discussion

The profiles of mean velocity and transient velocity are shown in figures 1-5. Fig. 1 exhibits the effects of Hartmann number, sink-strength and Prandtl number on the mean velocity. It is observed

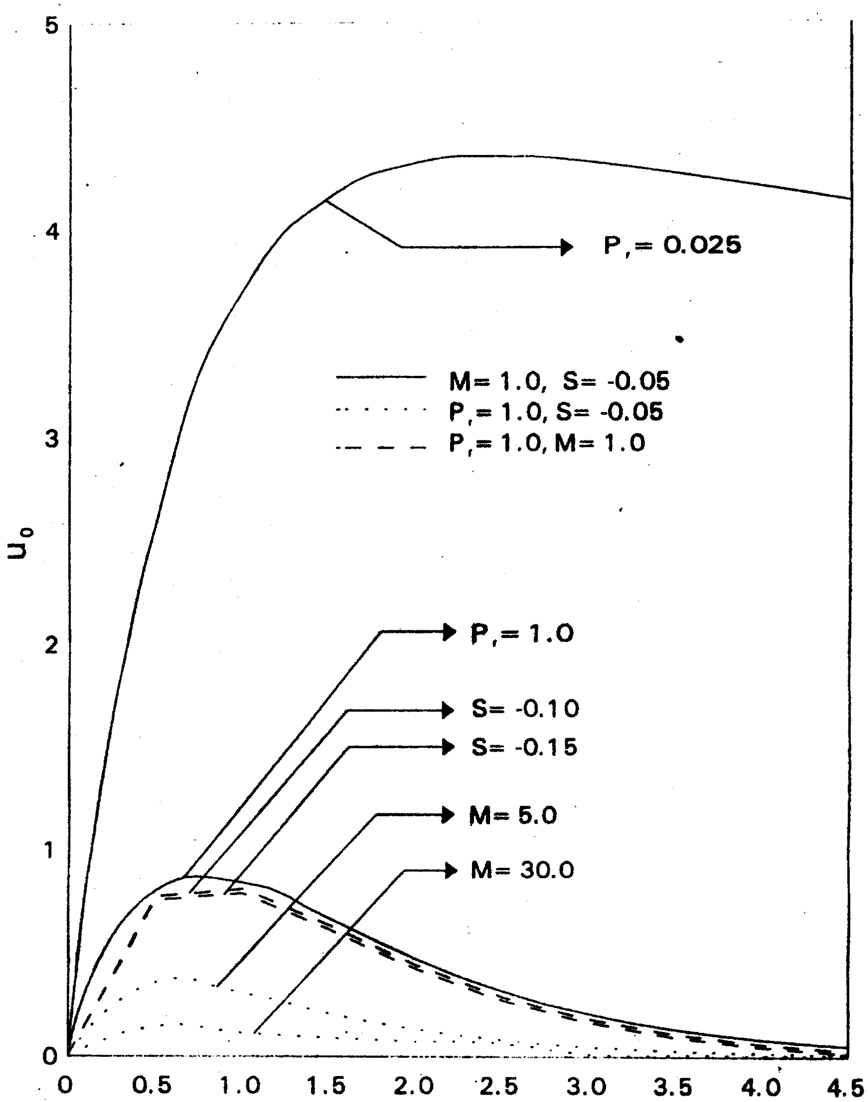

FIG. 1. Effects of P_r, R and S on mean velocity for G = 5.0, E_c = 0.001, ω = 5.0, ε = 0.2, $\omega t = \pi/2$

that increase in external magnetic field strength reduces the mean velocity and similar effect is marked in increasing the sink-strength. Also from Fig. 1, it is observed that the mean velocity is greater for mercury ($P_r = 0.025$) than that of electrolytic solution ($P_r = 1.0$). The effect of Hartmann number, sink strength and Prandtl number on transient velocity (u) are shown in Fig. 2 with other parameters are fixed. It is observed that, the transient velocity decreases with the increase in magnetic

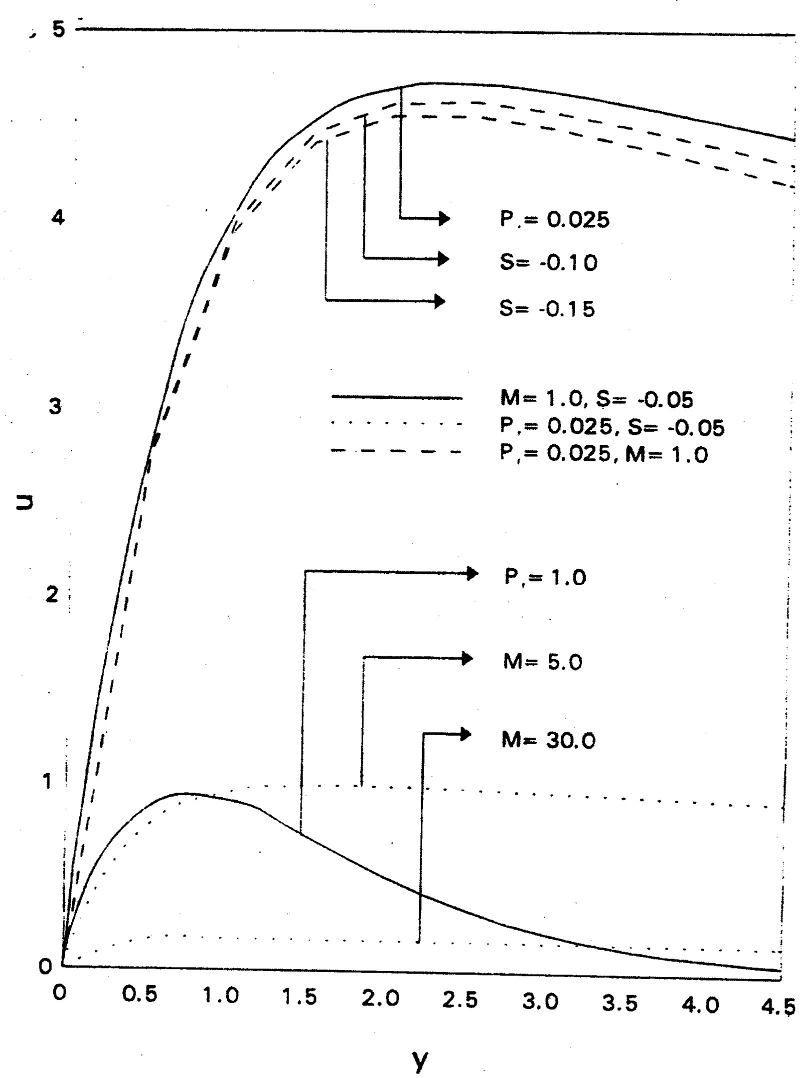

FIG. 2. Effects of P_r, M and S on transient velocity for G = 5.0, E_c = 0.001, ω = 5.0, ε = 0.2, $\omega t = \pi/2$

field strength. Consequently, the imposition of the external magnetic field decelerates the motion of the fluid. Also the transient velocity decreases with increase in sink-strength. Also it is found that the effect of Prandtl number is same as in case of mean velocity. Fig. 3 displays the effects of M and S on the mean temperature. It is marked that the increase in Hartmann number (M) reduces the

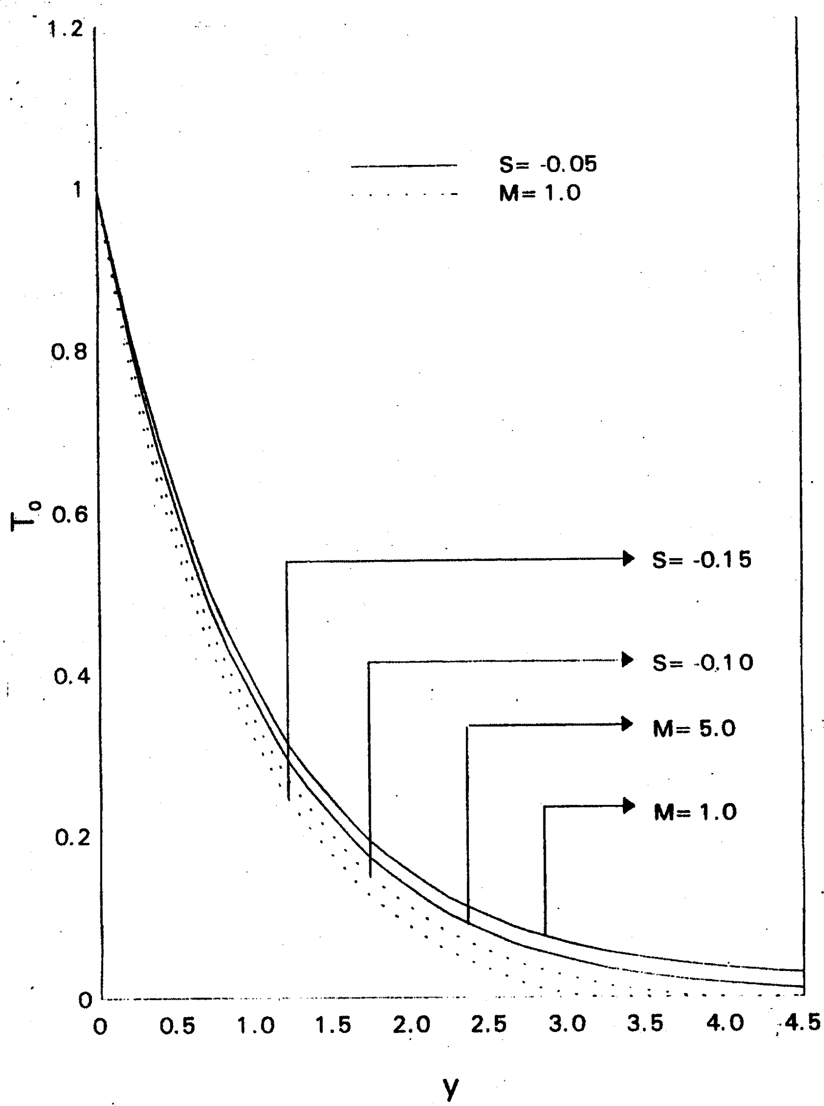

FIG. 3. Effects of M and S on mean temperature for G = 5.0, P_r = 1.0, E_c = 0.001, ω = 5.0, ε = 0.2, $\omega t = \pi/2$

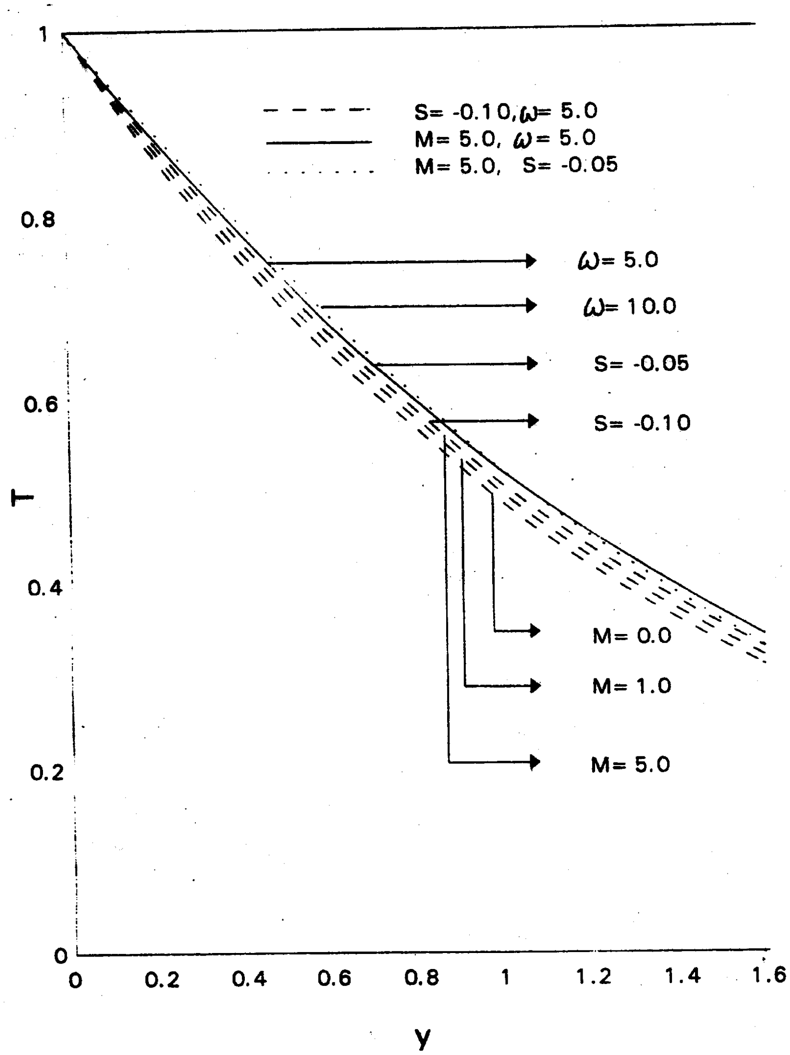

FIG. 4. Effects of M, S and ω on transient temperature for G = 5.0, P_r = 1.0, E_c = 0.001, ω = 5.0, ε = 0.2, $\omega t = \pi/2$

mean temperature. As the sink strength decreases, the mean temperature increases significantly. The effects of M, S and ω on the transient temperature have been exhibited by the curves shown in Fig. 4. It is observed that the increase in Hartmann number increases the transient temperature of the fluid. The increase in sink-strength, decreases the transient temperature. The effect of increase in

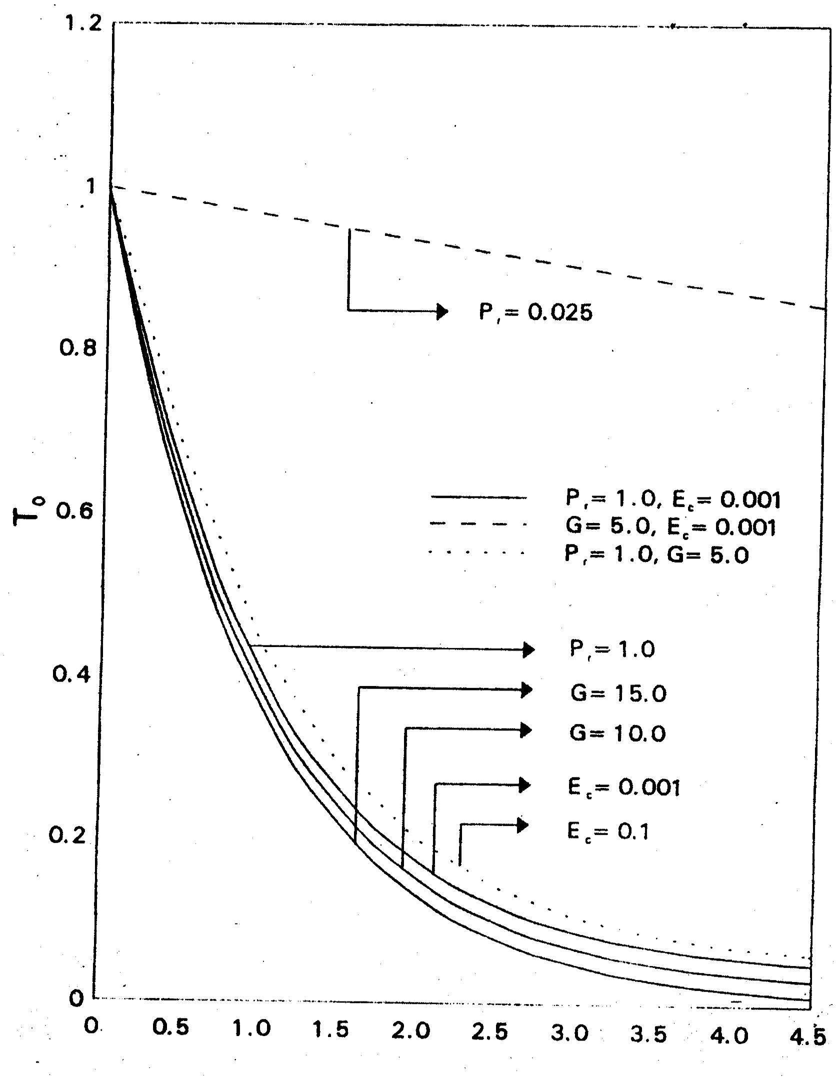

FIG. 5. Effects of P_r, G and E_c on mean temperature for $M = 1.0$, $S = -0.05$, $\omega = 5.0$, $\varepsilon = 0.2$, $\omega t = \pi/2$

frequency parameter(ω) is similar to that of increase in sink strength. The effects of P_r, G and E_c on mean temperature (T_0) are shown in Fig. 5.

It is observed that increase in Grashof number, thereby heating the plate, increases the mean temperature. Also increase in Eckert number raises the fluid temperature. It is found that the mean temperature is more for mercury (P_r = 0.025) in comparison to electrolytic solution (P_r = 1.0).

The mean skin-friction, amplitude and phase for mercury and electrolytic solution are given in Table 1. It is noticed that the increase in magnetic field strength decreases the mean skin-friction, amplitude and phase for both mercury and electrolytic solution. The mean rate of heat transfer and corresponding amplitude and phase are given in Table 2. It is observed that the mean rate of heat transfer decreases with increase in magnetic field strength or sink strength for both mercury and electrolytic solution. The amplitude decreases with the increase in magnetic field strength and increases with the increase in sink strength. However, as compared to the amplitude, reverse effect is observed in phase for both mercury and electrolytic solution.

TABLE 1:

Values of mean skin friction (τ_ω^m), amplitude ($|B|$), phase ($\tan \alpha$) for G = 5.0, E_c = 0.001, ω = 5.0, ε = 0.2, wt = $\pi/2$

P_r	M	S	τ_ω^m	$\lvert B \rvert$	$\tan \alpha$
Mercury (P_r = 0.025)	1.0	–0.05	7.6674	2.2235	–0.8394
	5.0	–0.05	2.7391	1.5777	–0.3194
	5.0	–0.10	2.7297	1.5762	–0.3180
Electrolytic solution (P_r = 1.0)	1.0	–0.05	3.0682	1.8391	–0.6547
	5.0	–0.05	1.7835	2.3038	–0.2207
	5.0	–0.10	1.7759	3.6557	–0.1194

TABLE 2:

Values of mean rate of heat transfer (q_ω^m), amplitude ($|H|$), phase ($\tan \gamma$) for G = 5.0, E_c = 0.001, ω = 5.0, ε = 0.2, wt = $\pi/2$

P_r	M	S	q_ω^m	$\lvert B \rvert$	$\tan \alpha$
Mercury (P_r = 0.025)	1.0	–0.05	-0.0336	1.2479	0.5636
	5.0	–0.05	–0.0341	1.2390	0.5752
	5.0	–0.10	–0.0404	1.2406	0.5705
Electrolytic solution (P_r = 1.0)	1.0	–0.05	-0.9330	1.5961	0.4999
	5.0	–0.05	–1.0089	1.8960	0.8966
	5.0	–0.10	–1.0209	1.8861	0.7321

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