# Dynamic Spectrum Allocation Algorithms for Industrial Cognitive Radio Networks 

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#### Abstract

Irregular spectrum usage and spectrum scarcity in emergency situations is a common problem in industrial wireless networks. To enhance the dynamic spectrum usage, cognitive radio network (CRN) is introduced in various automotive industrial wireless applications named as industrial cognitive radio network (ICRN). However, establishing the control channel by using channel hopping mechanism in ICRN is a challenging problem. In order to achieve reliable performance in ICRN, efficient channel hopping protocols need to be designed. In this paper, two channel hopping protocols are designed for the ICRN with or without the global clock synchronization to maximize the degree of rendezvous within the shortest time, minimize the inter rendezvous intervals and to reduce the maximum time to rendezvous by two secondary users. Performance evaluation of our protocols outperform in terms of throughput, percentage of rendezvous and average time to rendezvous over existing CRN protocols.


Index Terms-Asynchronous, channel hopping (CH), cognitive radio networks (CRNs), symmetric, synchronous.

## I. INTRODUCTION

COGNITIVE radio network (CRN) is introduced by Federal Communications Commission [1] to acknowledge the spectrum scarcity problems and to enhance the dynamic spectrum access (DSA) [2]. The main objective of CRN is to introduce some unlicensed users known as secondary users (SUs) to utilize the unused spectrum in the absence of the licensed users known as primary users (PUs) [3], [4]. Each SU is equipped with one or more cognitive radios [5], [6] and is capable of identifying the availability of channels not occupied by the PUs. The

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entities of a CRN are the SUs, which can adequately sense the spectrum holes to rendezvous with other SUs and establish the common control channel for transmitting data. Each SU executes the channel hopping $(\mathrm{CH})$ approach for the purpose of rendezvous to use the underutilized spectrum efficiently.

Wireless technologies, such as ZigBee, WiFi, Bluetooth, and 5G/LTE play an important role in various industrial applications in process controlling, monitoring, surveillance, and other connected embedded systems. In such technological environment of industrial wireless networks, multiple heterogeneous wireless devices may operate in a frequency band utilized by many other wireless technologies, which are usually harsh and time varying. Based on the current regulation of radio spectrum for the industrial wireless devices, the spectrum assignment is fixed for which devices cannot switch to other spectrum even if it is congested [7] at particular regions. Besides, some part of the allocated spectrum in industry are overutilized, whereas some part of it are underutilized, which creates spectrum holes based on time and space. In order to mitigate such problems, cognitive radio technology should be integrated to the industrial wireless devices for spectrum sensing and DSA, which is called industrial cognitive radio network (ICRN).

The fire alarm notification in industries requires quick delivery of messages. If the associated licensed spectrum is not available at that time, the entire factory floor might be under fire. Due to scarcity of the communication channels, if the data are received at wrong instance of time, the situation might lead to inaccurate decision of monitoring system [3]. Considering the harsh and emergency environment of the industries, ICRN needs to be embedded with much hardened and smarter cognitive antennas and devices as compared to the devices of general CRN. ICRN can use the cognitive radios to sense the spectrum holes in the emergency situations to transmit data. In an industry, normally WiFi and cellular network are busy and are overutilized, whereas the spectrum assigned for the satellite communication is underutilized. In this scenario, concept of CRN can be used in industries by introducing the unlicensed/secondary WiFi or cellular users to use the vacant portion of the satellite spectrum for emergency data communication.

In wireless networks, though CH multichannel protocols can be used to establish the common control channel for the medium access, the mechanism is quite different from the CRN in terms of operational and sensing capability. Unlike CRN, the wireless communication devices do not have cognitive radios to sense the unused spectrum. Moreover, the devices in wireless net-
work operate on Industrial, Scientific and Medical (ISM) bands and there is no distinction between the devices as primary or secondary as all devices have equal rights over the channels to access. However, in CRN the incumbent users operated over the licensed spectrum are PUs and always have the first priority to use the channels. Each SU executes the CH approach in absence of the PUs, where it needs to hop from one channel to another to find the common vacant channel for making their rendezvous successful [8], [9].

SUs have to preempt the channels as soon as a PU returns to use it and the SU has to hop to another channel to find out the unused channel at different time instance [10]. The primary challenge in the CH approach is how to increase the degree of rendezvous, including minimization of the maximum time to rendezvous (MTTR) between SUs [11]-[13]. In this paper, symmetric synchronous (SymSyn) CH and symmetric asynchronous (SymAsyn) CH protocols are proposed to maximize the degree of rendezvous as well as to minimize MTTR with maximum channel utilization. It is to be noted that throughout this paper, CRN is alternatively used to represent ICRN.

Rest of the paper is organized as follows. Related works with our motivation and contributions are given in Section II. System model of the proposed CH protocol is given in Section III. The proposed symmetric synchronous and asynchronous CH protocols are designed in Section IV. Performance evaluation of our protocols is given in Section V and concluding remarks are made in Section VI.

## II. Related work

In CH approach, network can be considered as synchronous or asynchronous and the channel type can be symmetric or asymmetricbased on the availability of channels for the SUs coexist with the PUs [11]-[13]. According to the definition of the synchronous environment, all SUs can enter into the network at the same instance of time, whereas in asynchronous scenario SUs can enter into the network at any point of time [14]-[16]. In symmetric CH approach, each SU can have same set of available channels located in the same geographical area, whereas in asymmetric CH approach, each SU can have different sets of available channels with at least one common channel among them [13], [17]. Wu et al. [12], propose PRICH (roundrobin indemnity- CH ) and CACH (cycle-adjustable CH ) in synchronous and symmetric environment. However, the degree of rendezvous in both protocols is very small, which is $N$ out of $N \times(N+1)$ time slots, where N is the number of available channels for SUs.

Chang et al. [1] propose rendezvous couple CH (RCCH) scheme, which can increase the channel utilization ratio, but the degree of rendezvous is very small, i.e., $N$ and the value of MTTR is large, i.e., $\frac{N}{2}$. In [1], another CH scheme is introduced as asynchronous rendezvous CH (ARCH) where probability of rendezvous is very less, i.e., $\frac{1}{N}$. Beside, MTTR is very large, which is $(2 N-1)$. In [11], Asyn-ETCH protocol is designed for the asynchronous environment. However, it can have small degree of rendezvous, i.e., $\lesssim 50 \%$ with large MTTR $\lesssim 2 N$.

In [14], A-QCH (asynchronous quorum based CH system) algorithm is proposed where the number of rendezvous channel
is always two. Beside, in the CH system A-MOCH [14], the degree of rendezvous is only $N$ out of $N^{2}$ time slots. The rendezvous is not uniform in each cycle and some of the cycles are over- or underutilized. In [15], enhanced alternate hop and wait (E-AHW ) CH algorithm is proposed for asynchronous symmetric environment. In E-AHW, average degree of rendezvous is only $15 \%$, which occurs only in a single channel.

According to [13], the MTTR between two SUs in SymAsyn scenario is $3 P$, where $P$ is a prime number and $P>N$. In other words, two SUs need to wait for an entire inner round to have the rendezvous. In [18], ID and non-ID-based T-CH and $\mathrm{D}-\mathrm{CH}$ protocols are proposed. In T-CH, though degree of rendezvous is increased, there is higher chance of collision in multiuser scenario as the sender and the receive use the same CH sequence. In D-CH, degree of rendezvous is very small with large value of MTTR, if both SUs have different IDs. In [9], although the authors have analyzed the feasible interference region for the PUs caused by the SUs in a CRN but there is no analysis about the percentage of spectrum usage and degree of rendezvous.

From the survey of all latest CH algorithms in CRN, it is observed that most of the proposed algorithms underperform in terms of degree of rendezvous, MTTR, maximum inter rendezvous interval (MIRI) and channel utilization. In this paper, efficient symmetric CH algorithms for the synchronous and asynchronous environment are designed and the main contributions of our work can be summarized as follows.

1) Our proposed protocols can work for any number of available channels.
2) The degree of rendezvous between any two SUs in our SymSyn CH Scheme is $\lesssim \frac{N^{2}}{2}$, which is highest as compared to the existing protocols.
3) In our SymSyn protocol, we guarantee the successful rendezvous between two SUs with shortest interval of time such that MTTR $\leq \frac{N}{4}$.
4) A SymAsyn CH Scheme is introduced in which average degree of rendezvous is more than $50 \%$ with MTTR $\leq \frac{3 N}{4}$.
5) The degree of rendezvous in our protocols increases with increase in the number of available channels, whereas this value normally decreases in case of other existing protocols.

## III. Problem Formulation

Consider a CRN with both licensed PUs and unlicensed SUs in which fixed number of licensed orthogonal channels are accessed by all PUs. Depending on the absence of the PUs, SUs can access the licensed channels opportunistically. For simplicity, let $N$ be the set of available channels for all SUs indexed from $\{0, \ldots,(|N|-1)\}$. Each SU is equipped with one halfduplex cognitive radio transceiver, which can have the capability to sense the spectrum holes. The entire network is divided into multiple slot indexes starting from $0,1, \ldots, T_{l}-1$, where $T_{l}$ $=$ length of the CH sequence. The CH pattern of a particular SU is determined by its CH sequence. The entire CH sequence of a SU comprises with two parameters, i.e., slot index, and


Fig. 1. Example of dynamic spectrum allocation in cognitive radio network (CRN).
channel number, which is defined as CH_seq $=\left(\mathrm{T}_{\mathrm{i}}, \mathrm{C}_{\mathrm{j}}\right), \forall \mathrm{i} \in$ $\left[0,\left(\mathrm{~T}_{1}-1\right)\right]$ and $\forall \mathrm{j} \in[0,(|\mathrm{~N}|-1)]$, where the value of $T_{l}>|N|$. Therefore, any channel $c \in C_{j}$ will appear more than once in the hopping sequence. The CH sequence can be expressed as CH_seq $=\left\{\left(0, c_{0}\right),\left(1, c_{1}\right), \ldots,\left(i, c_{i}\right), \ldots\right.$, $\left.\left(T_{l}-1, c_{T_{l}-1}\right)\right\}$, where $c_{i} \in[0,|N|-1]$ represents the channel number visited by a SU at i-th slot index in a CH sequence. If two SUs hop to the same channel at the same slot index, they can listen to each other and use this channel as an operational channel for their data communication. Let CH 1 and CH 2 be the CH sequences of SU1 and SU2, respectively. The channel $c$ can be the common channel for both the SUs, if $\exists i \in\left[0, T_{l}-1\right]$, such that $(i, c) \in(\mathrm{CH} 1 \cap \mathrm{CH} 2)$. Let us consider a CRN that comprises both licensed PUs and unlicensed SUs with $|N|$ number of available channels. Considering an example as shown in Fig. 1, let number of available channels be five, i.e., $N=\{0,1,2,3,4\}$. The entire network is divided into multiple slot indexes indexed from $\{0,1,2,3,4\}$. From Fig. 1, it can be observed that some of the channels such as channel 1, 2, 3, and 4 are underutilized by the PUs at certain slots, thereby creating the spectrum holes. Hence, unlicensed SUs are embedded with smarter cognitive antenna to utilize the vacant portion of the channels more efficiently. Each SU follows a predefined CH sequence to hop from one channel to another and makes the rendezvous successful, as shown in Fig. 1.

## A. Preliminaries

1) Degree of rendezvous: This metric is defined as the number of times two SUs meet with each other within a CH sequence.
2) Maximum time to rendezvous (MTTR): MTTR is the maximum time slots a SU takes to rendezvous within a hopping sequence. It is the worst case of TTR (time to rendezvous).
3) Average time to rendezvous (ATTR): Average time to rendezvous between two SUs is calculated as the average time required for SUs to rendezvous in one cycle. Hence, average of all possible TTRs is taken into account to calculate the ATTR.
4) Maximum inter rendezvous interval (MIRI): The MIRI is the maximum number of slot indexes between any two consecutive rendezvous between two SUs in a CRN. The

MIRI is defined as the $\operatorname{MAX}\left\{\left(t_{j}-t_{i}\right)-1\right\}$ (where $t_{i}$ and $t_{j}$ are the slot indexes of two consecutive rendezvous). The value of MIRI should be minimized to reduce the waiting period between the first and the next rendezvous.

## B. System Model

In this section, we introduce a distributed mechanism to generate a general sequence set, as given in Algorithm 1, which is common to our proposed symmetric synchronous and asynchronous environment. In our protocols, CH sequence comprises with a set of general sequences denoted as GS $=\left\{s_{0}\right.$, $\left.s_{1}, s_{2}, \ldots, s_{m-1}\right\}$, where $m<|N|$. SUs generate $m$ number of general sequences by taking $|N|$ number of available channels into consideration. As given in Algorithm 1, first a pivot element is selected from the set of the available channels, which is then divided into three sets of channels as $N_{\text {front }}\left(N_{f}\right), N_{\text {middle }}$ ( $N_{m}$ ), and $\mathrm{N}_{\text {back }}\left(\mathrm{N}_{b}\right)$ with help of that pivot element. A set of channel-shifting seeds $(r)$ is obtained from the set $N_{f}$ and $N_{b}$ separately based on the rules given in the Algorithm 1. Then, the channel-shifting seeds are used for shifting the channels to generate the updated sets $N_{f}$ and $N_{b}$. Finally, the general sequence set GS $=\left\{s_{0}, s_{1}, \ldots, s_{m-1}\right\}$ is generated by calling the function sequence_geneartion as given in Algorithm 2 and is obtained by concatenating the updated sets $N_{f}$ and $N_{b}$ with $N_{m}$.

The generation of general sequences, as given in Algorithm 1, can be explained as follows. In line 2 , a pivot element $p$ is selected, whereas assignment of channels to set $N_{f}, N_{m}$, and $N_{b}$ is made as given in lines 3 through 5 . In lines 6 through 10, channelshifting seed set $r$ is calculated depending on the cardinality of $\left|N_{f}\right|$. In line 12, another sub function (sequence_generation) is called that returns the set of general sequences generated from the set $N_{f}$. The channel-shifting seeds for set $N_{b}$ is calculated as given in lines 14 through 18. In line 20, again intermediate function (sequence-generation) is called to generate the general sequences from the set $N_{b}$. In line 21, both sets $S_{N_{f}}$ and $S_{N_{b}}$ are united to generate the final set of general sequence (GS). The set of general sequences GS that contains all generated general sequences is presented in line 22 . In order to separate the generation of individual general sequences, a parameter sequence_bit is used. The channels of set $N_{f}$ are executed if the sequence_bit $==0$ and the set $N_{b}$ is executed if the sequence_bit $==1$.

As presented in Algorithm 2, the function sequence_generation represents all shifting and movements of the channels present in the set $N_{f}$ and $N_{b}$. The elements of set $N_{f}$ and $N_{b}$ are assigned to the set $N_{a}$ depending on the value of sequence_bit as given in lines 2 through 7 . Lines 8 through 41 show how the general sequences are generated after applying each channel-shifting seed to both the set $N_{f}$ and $N_{b}$. If channel-shifting seed is 0 , all elements of set $N_{a}$ are assigned to the intermediate set $M_{i}$ as presented in lines 10 through 11. In line 13, calculation of shifting_channel is done. If channelshifting seed is not 0 , the codes given in lines 12 through 32 are executed. The lines 12 through 32 represent the anticlockwise movement of the channels after applying individual channel shifting seeds. The resulting channels generated due to anti-

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Algorithm 1: General sequence generation.
Input: \(N\) : Set of available channels in the CRN.
Output: \(G S\) : Set of general sequences.
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## Notations:

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\(N_{f}\) : Set that stores front segment of channels of set \(N\);
\(N_{b}\) : Set that stores back segment of channels of set \(N\);
\(N_{m}\) : Set that stores the pivot channel only;
sequence_bit : Sequence bit that signifies the execution of set \(N_{f}\) or \(N_{b} ; r\) : Set that stores the channel-shifting seeds; \(p\) : pivot_element; \(S_{N_{f}}, S_{N_{b}}\) : Sets that stores generated general sequences temporarily from set \(N_{f}\) and \(N_{b} ; N[]\) : An array of channels.
Initialize \(N_{f}=N_{m}=N_{b}=r=G S=\) \(S_{N_{f}}=S_{N_{b}}=\phi\);
    \(p=\lfloor|N| / 2\rfloor\);
    \(N_{m}=p\) th element of set \(N=\{N[p-1]\}=\)
    pivot channel;
    \(N_{f}=\{0,1, \ldots, N[p-2]\}\);
    \(N_{b}=\{N[p], \ldots,|N|-1\} ;\)
    if \(\left(\left|N_{f}\right| \% 2==0\right)\) then
            \(r=\left\{0,2,4, \ldots,\left(\left|N_{f}\right|-2\right)\right\} ;\)
        else
            \(r=\left\{0,1,3, \ldots,\left(\left|N_{f}\right|-2\right)\right\} ;\)
        end if
        sequence_bit \(=0 ; / /\) Is taken for \(N_{f}\)
        \(S_{N_{f}}=\) sequence_generation \(\left(N, N_{f}, N_{m}, N_{b}, r\right.\),
        sequence_bit);
    \(r=\phi ;\)
    if \(\left(\left|N_{b}\right| \% 2==0\right)\) then
        \(r=\left\{0,2,4, \ldots,\left(\left|N_{b}\right|-2\right)\right\} ;\)
    else
            \(r=\left\{0,1,3, \ldots,\left(\left|N_{b}\right|-2\right)\right\} ;\)
        end if
        sequence_bit \(=1\); //Is taken for \(N_{b}\)
    \(S_{N_{b}}=\) sequence_generation \(\left(N, N_{f}, N_{m}, N_{b}, r\right.\),
        sequence_bit);
    \(G S=S_{N_{f}} \cup S_{N_{b}} ;\)
        Return \(G S\)
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clockwise movement are stored in the intermediate set $M_{i}$. General sequences are generated by combining the channels present in the set $N_{f}, N_{b}, N_{m}$ and $M_{i}$ depending on the sequence_bit as given in lines 33 through 38. In line 39, each generated general sequence is concatenated with the temporarily stored sequence $S^{\prime}$. In line 42, the generated general sequences stored in $S^{\prime}$ are returned to the main function general_sequence_generation.

Let us consider an example, where $|N|=9$ is the number of available channels for the SUs in a CRN. Hence, set of available channels can be indexed as $N=\{0,1, \ldots, 8\}$, which can be initially arranged clock-wise in a ring as shown in Fig. 2(a). Then, the pivot element $p$ is calculated based on our algorithm, which can be $p=\left\lfloor\frac{9}{2}\right\rfloor=4$ here. Thus, set $N_{m}$ contains only the pivot channel i.e. $N_{m}=p$-th element of set $\mathrm{N}=4$ th element of

```
Algorithm 2: sequence_generation function call.
Input: \(N, N_{f}, N_{m}, N_{b}, r\), sequence_bit;
Output: \(S^{\prime}\) : Set that stores the general sequences
temporarily generated from sets \(N_{f}\) and \(N_{b}\).
Notations:
\(N_{a}\) : Set that stores elements of \(N_{f}\) or \(N_{b}\) temporarily
depending on the values of sequence_bit; \(M_{i}\) : Set that
stores the intermediate sequences after using
channel-shifting seeds; initial_channel : Stores the
first element of set \(N_{a}\); shifting_channel : Stores the
shifting channel after applying channel shifting seeds;
\(r[]\) : An array of channel-shifting seeds used to access
individual channel-shifting seeds.
Initialize \(N_{a}=M_{i}=s=S^{\prime}=\phi\);
if sequence_bit \(==0\) then
Assign all the elements of set \(N_{f}\) to \(N_{a}\) i.e.,
\(N_{a}=N_{a} \cup N_{f} ;\)
end if
if sequence_bit \(==1\) then
Assign all the elements of set \(N_{b}\) to \(N_{a}\), i.e.,
\(N_{a}=N_{a} \cup N_{b} ;\)
    end if
    initial_channel \(=N_{a}[0] ;\) len \(_{N_{a}}=\left|N_{a}\right|\);
    \(l e n_{N_{f}}=\left|N_{f}\right| ;\) len \(_{N_{b}}=\left|N_{b}\right| ;\)
    for \(i=0\) to \((|r|-1)\) do
        if \(r[i]==0\) then
            Assign all elements of set \(N_{a}\) to \(M_{i}\) i.e.,
            \(M_{i}=M_{i} \cup N_{a} ;\)
        else
            shifting_channel \(=\)
            \(\left[\right.\) initial_channel \(\left.+l e n_{N_{a}}\right]-r[i]\);
            Append the shifting_channel to set \(M_{i}\), i.e.,
            \(M_{i}=M_{i} \cup\) shifting_channel;
            shifting_channel \(=\) shifting_channel -1 ;
            while shifting_channel \(>=\) initial_channel
            do
                Append the shifting_channel to set \(M_{i}\) i.e.,
                \(M_{i}=M_{i} \cup\) shifting_channel;
                    \(l e n_{N_{a}}=l e n_{N_{a}}-1\);
                    shifting_channel \(=\) shifting_channel -1 ;
            end while
            while \(l e n_{N_{a}} \neq 1\) do
                    if sequence_bit \(==0\) then
                \(l e n_{N_{f}}=l e n_{N_{f}}-1\);
                Append the \(l e n_{N_{f}}\) th element of set \(N_{f}\) to
                set \(M_{i}\) i.e. \(M_{i}=M_{i} \cup N_{f}\left[l e n_{N_{f}}\right]\);
            end if
            if sequence_bit \(==1\) then
                \(l e n_{N_{b}}=l e n_{N_{b}}-1\);
                Append the \(l e n_{N_{b}}\) th element of set \(N_{b}\) to set
                \(M_{i}\) i.e. \(M_{i}=M_{i} \cup N_{b}\left[l e n_{N_{b}}\right] ;\)
            end if
            \(l e n_{N_{a}}=l e n_{N_{a}}-1\);
            end while
            end if
```

| Continoud. |  |
| :--- | :---: |
| 33: | if sequence_bit $==0$ then |
| 34: | $s=\left\{M_{i} \cup N_{m} \cup N_{b}\right\} ;$ |
| 35: | end if |
| 36: | if sequence_bit $==1$ then |
| 37: | $s=\left\{N_{f} \cup N_{m} \cup M_{i}\right\} ;$ |
| 38: | end if |
| 39: | Concatenate each generated general sequence with |
| 40: | the general sequence set $S^{\prime}=S^{\prime} \\| s ;$ |
| 41: | end for $=s=\phi$ |
| 42: | Return $S^{\prime}$ |




Fig. 3. Generation of general sequences from set $N_{b}$.
contains channels $\{4,5,6,7,8\}$. Since, $\left|N_{b}\right|=5, r=\{0,1,3\}$, as $\left(\left|N_{b}\right|-2\right)=3$. After applying each channel-shifting seed on the channels present in the set $N_{b}$, three new sets of $N_{b}$ are generated as $N_{b}=\{4,5,6,7,8\}$ for channel-shifting seed $=0$, as shown in Fig. 3(a), $N_{b}=\{8,7,6,5,4\}$ for channel-shifting seed $=1$, as shown in Fig. 3(b), and $N_{b}=\{6,5,4,8,7\}$ for channel-shifting seed $=3$, as shown in Fig. 3(c). Finally, general sequences are generated by performing the union operations between the channels of set $N_{f}=\{0,1,2\}$ and set $N_{m}$ with each newly generated set $N_{b}$, as shown in Fig. 3(d). In summary, by using Algorithms 1 and 2 and as shown in Fig. 2, general sequences $s_{0}$ and $s_{1}$ are generated and as shown in Fig. 3, general sequences $s_{2}, s_{3}$, and $s_{4}$ are generated. Thus, for $|N|=9$ as the number of available channels, 5 number of general sequences are generated and the set of general sequences (GS) can be GS $=\{\langle 0,1,2,3,4,5,6,7,8\rangle,\langle 2,1,0,3,4,5,6,7,8\rangle,\langle 0,1,2,3$, $4,5,6,7,8\rangle,\langle 0,1,2,3,8,7,6,5,4\rangle,\langle 0,1,2,3,6,5,4,8,7\rangle\}$.

## iV. Optimal Rendezvous CH Protocols

In this section, SymSyn and SymAsyn CH protocols are proposed, which can achieve optimal value in terms of degree of rendezvous, MTTR, ATTR, and MIRI. Detail algorithms of these protocols are described as follows.

## A. SymSyn CH Protocol

Based on the standard definition of symmetric synchronous environment of CRN, each SU enters into the CRN at the same instant of time with availability of $N$ licensed channels indexed from 0 to $|N|-1$. The slot boundaries of all SUs are assumed to be aligned and the CH is started from the initial slot index 0 . As described in the previous section, the general sequences $s_{0}, s_{1}, \ldots, s_{m-1}$ are generated, which are then shared by all SUs to generate the CH sequence. The algorithm for generating the CH sequence is given in Algorithm 3.

Let $|N|=9$ be the number of available channels and GS $=$ $\left\{s_{0}, s_{1}, s_{2}, s_{3}, s_{4}\right\}$ be the set of general sequences generated by two SUs, SU1 and SU2 by executing Algorithms 1 and 2. Then, each SU creates its own CH sequence by randomly choosing


Fig. 4. Rendezvous between two secondary users (SUs) in symmetric synchronous environment.

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Algorithm 3: Generation of CH sequence for SymSyn Pro-
tocol.
Input: \(N\) : Set of available channels in the CRN.
Output: CH_seq: Channel Hopping sequence.
    Notations
    \(G S\) : Set of general sequences;
    Hopping Sequence \((H S)\) : Set that stores permuted
        order of general sequences.
        Initialize \(H S=G S=\mathrm{CH} \_\)seq \(=\phi\);
        \(G S=\) general_sequence_generation \((N)\);
        \(H S=H S \cup\{\operatorname{perm}(G S)\} ;\)
        CH_seq \(=<C H \_\)seq \(\| H S>\);
```

any one of the permutation of general sequences from the set GS. Let, $s_{0}, s_{1}, s_{2}, s_{3}$, and $s_{4}$ be the general sequences of SU1. As shown in Fig. 4, the general sequences of SU1 are $s_{0}$ : $\langle 0,1,2,3,4,5,6,7,8\rangle, s_{1}:\langle 2,1,0,3,4,5,6,7,8\rangle, s_{2}:\langle 0,1$, $2,3,4,5,6,7,8\rangle, s_{3}:\langle 0,1,2,3,8,7,6,5,4\rangle, s_{4}:\langle 0,1,2,3,6$, $5,4,8,7\rangle$. Let, $s_{1}, s_{0}, s_{2}, s_{3}$, and $s_{4}$ be the general sequences of SU2. As shown in Fig. 4, the general sequences of SU2 are $s_{1}:\langle 2,1,0,3,4,5,6,7,8\rangle, s_{0}:\langle 0,1,2,3,4,5,6,7,8\rangle, s_{2}$ : $\langle 0,1,2,3,4,5,6,7,8\rangle, s_{3}:\langle 0,1,2,3,8,7,6,5,4\rangle, s_{4}:\langle 0$, $1,2,3,6,5,4,8,7\rangle$. In this example, it can be observed that the degree of rendezvous between both SUs in a CH sequence is 41 out of 45 number of slot indexes. Then, the performance metrics of our proposed SymSyn CH protocol can be analyzed as follows.

1) Degree of Rendezvous: In our (SymSyn) CH Protocol, rendezvous occurs either in the channels of the set $N_{f}$ (when channels of set $N_{b}$ go for the shifting) or in the set $N_{b}$ (when channels of set $N_{f}$ go for the shifting) and there must be a guaranteed rendezvous among the channels present in the set $N_{m}$. Therefore, there must be rendezvous between two SUs.

Lemma 1: The maximum degree of rendezvous is $\leq$ $\left[\{(m-2) \times|N|\}+\left\{2 \times\left(c_{f}+\left|N_{m}\right|+\left|N_{b}\right|\right)\right\}\right]$ for $|N| \geq 8$, else maximum degree of rendezvous is $\leq[\{(m-2) \times|N|\}+$ $\left.\left\{2 \times\left(\left|N_{f}\right|+\left|N_{m}\right|+c_{b}\right)\right\}\right]$, when both SUs permute different sets of general sequences. Value of $c_{b}$ and $c_{f}$ could be 1 or 2 based on the value of $\left|N_{b}\right|$ and $\left|N_{f}\right|$ is odd or even.

Proof: Let $m$ number of general sequences be generated from $|N|$ number of available channels by SU1 and SU2. Let $\mathrm{PS} 1=\left\{s_{i}, s_{j}, s_{1}, s_{0}, \ldots, s_{m-1}\right\}$ and $\mathrm{PS} 2=\left\{s_{j}, s_{i}, s_{1}, s_{0}, \ldots\right.$, $\left.s_{m-1}\right\}$ be the permutation of general sequences chosen by SU1
and SU2, respectively. From these chosen general sequences, it can be observed that the order of general sequence from $s_{1}$ to $s_{m-1}$ is same for both SUs, whereas the order of $s_{i}$ and $s_{j}$ is swapped between both permuted sets. It is known that the CH sequence of a $\mathrm{SU}=$ the permuted general sequence set. Hence, the degree of rendezvous between the general sequences of PS1 and PS2 starting from $s_{1}$ to $s_{m-1}$ is $(m-2) \times|N|$. Now, we need to analyze how many degree of rendezvous can occur within the first two general sequences of PS1 and PS2, i.e., $\left\{s_{i}, s_{j}\right\}$ and $\left\{s_{j}, s_{i}\right\}$. It can be calculated that the degree of rendezvous in both $\left\{s_{i}, s_{j}\right\}$ and $\left\{s_{j}, s_{i}\right\}$ are $\left(c_{f}+\left|N_{m}\right|+\left|N_{b}\right|\right)$ when $\left|N_{f}\right|$ is even and odd, respectively, where $c_{f}=1$, if $\left|N_{f}\right|$ is odd and 2, if $\left|N_{f}\right|$ is even. Considering all degrees of rendezvous, it can be calculated that maximum degree of rendezvous $=\left[\{(m-2) \times|N|\}+\left\{2 \times\left(c_{f}+\right.\right.\right.$ $\left.\left.\left.\left|N_{m}\right|+\left|N_{b}\right|\right)\right\}\right]$. Similarly, for $|N|<8$, the maximum degree of rendezvous depends upon the general sequences generated from set $N_{b}$. Hence, in this scenario, the maximum degree of rendezvous is $\left[\{(m-2) \times|N|\}+\left\{2 \times\left(\left|N_{f}\right|+\left|N_{m}\right|+c_{b}\right)\right\}\right]$ where $c_{b}=1$, if $\left|N_{b}\right|$ is odd and 2, if $\left|N_{b}\right|$ is even. For different set of permutations, degree of rendezvous could be $<\left[\{(m-2) \times|N|\}+\left\{2 \times\left(c_{f}+\left|N_{m}\right|+\left|N_{b}\right|\right)\right\}\right]$ or $[\{(m-$ 2) $\left.\times|N|\}+\left\{2 \times\left(\left|N_{f}\right|+\left|N_{m}\right|+c_{b}\right)\right\}\right]$ depending on the number of channels.

Let us consider an example to explain degree of rendezvous. Let GS $=\left\{s_{0}, s_{1}, s_{2}, s_{3}, s_{4}\right\}$ be the set of general sequences generated when $|N|=9$, where $s_{0}$ and $s_{1}$ are generated from the set $N_{f}$ and $s_{2}, s_{3}$, and $s_{4}$ are generated from the set $N_{b}$. Let $\left\{s_{0}, s_{1}, s_{4}, s_{2}, s_{3}\right\}$ and $\left\{s_{1}, s_{0}, s_{4}, s_{2}, s_{3}\right\}$ be the permuted sets of general sequences chosen by SU1 and SU2, respectively. Here, SU1 and SU2 have the same order of permuted general sequences except $s_{0}$ and $s_{1}$, which are swapped. Hence, the maximum degree of rendezvous $=[\{(m-2) \times|N|\}+\{2 \times$ $\left.\left.\left(c_{f}+\left|N_{m}\right|+\left|N_{b}\right|\right)\right\}\right]=[\{(5-2) \times 9\}+\{2 \times(1+1+5)\}]$ $=41$, where $\left|N_{b}\right|=5$.
2) Average Time To Rendezvous (ATTR) and Maximum Time To Rendezvous (MTTR): The value of TTR varies with the ways channel-shifting seeds are used in the $N_{f}$ or $N_{b}$ part of a general sequence. If the initial general sequence of both SUs belongs to the general sequence generated by applying the channel-shifting seed on set $N_{b}$, then TTR $=1$ as the channels present in the set $N_{f}$ are same for all the general sequences generated from the set $N_{b}$. However, if the initial general sequences of both SUs are generated from the set $N_{f}$, the TTR varies with the of $\left|N_{f}\right|$ and the values of the channel-shifting seeds.

Lemma 2: If two SUs generate their first general sequence from the set $N_{f}$, one SU with channel-shifting seed $=0$ and other SU with a nonzero channel-shifting seed, then ATTR $=$ $\frac{\frac{|N|}{4}+2}{2}$ and MTTR $=\frac{|N|}{4}$.

Proof: Let us consider a set of available channels such that $|N|=16$, where the channels present in the set $N_{f}, N_{b}$, and $N_{m}$ are $\{0,1,2,3,4,5,6\},\{8,9,10,11,12,13,14,15\}$, and $\{7\}$, respectively. For the analysis of ATTR and MTTR, only the channels of set $N_{f}$ are considered. Let FS be the set of all general sequences generated from the set $N_{f}$. Hence, FS $=\left\{s_{0}:\langle 0, \mathbf{1}\right.$, $\mathbf{2}, \mathbf{3}, 4,5,6,7,8,9,10,11,12,13,14,15\rangle, s_{1}:\langle 6,5,4, \mathbf{3}$, $2,1,0,7,8,9,10,11,12,13,14,15\rangle, s_{2}:\langle 4,3,2,1,0,6$, $5,7,8,9,10,11,12,13,14,15\rangle, s_{3}:\langle 2,1,0,6,5,4,3,7,8$, $9,10,11,12,13,14,15\rangle\}$. It is to be noted that general sequence $s_{0}$ is generated from the channel-shifting seed 0 , while general sequences $s_{1}, s_{2}, s_{3}$ are generated from the nonzero channel-shifting seeds. Let SU1 considers the general sequence $s_{0}$ and SU2 considers either one of the general sequences $s_{1}$, $s_{2}$, or $s_{3}$. Hence, combinations of the general sequences could be $\left\{s_{0}, s_{1}\right\},\left\{s_{0}, s_{2}\right\}$, and $\left\{s_{0}, s_{3}\right\}$ and from the above combinations, TTR could be $2,3, \ldots, \frac{|N|}{4}$. Thus, the average TTR can be calculated as ATTR $=\frac{\left(2+3+\ldots+\frac{|N|}{4}\right)}{\frac{|N|}{4}-1}=\frac{\frac{\left(\frac{|N|}{4}-1\right) \times\left(\frac{|N|}{4}+2\right)}{\frac{|N|}{4}-1}}{\frac{\mid(N \mid}{4}+2} \frac{2}{2}$. In this case, maximum TTR is the maximum value of all TTRs between SUs, i.e., MTTR $=\operatorname{MAX}\left(2,3, \ldots, \frac{|N|}{4}\right)=\frac{|N|}{4}$. Particularly, if the combination of both SUs is $\left\{s_{0}, s_{1}\right\}$, then both SUs need to wait for four time slots, which is $\frac{|N|}{4}$ and it can also be true for any number of channels.

Lemma 3: If two SUs generate their first general sequence from the set $N_{f}$ by using any nonzero channel-shifting seed, ATTR $=\frac{|N|}{2}$ and MTTR $=\frac{|N|}{2}$.

Proof: Let $s_{1}:\langle 6,5,4,3,2,1,0,7,8,9,10,11,12,13,14$, $15\rangle, s_{2}:\langle 4,3,2,1,0,6,5,7,8,9,10,11,12,13,14,15\rangle$ be the initial general sequences of SU1 and SU2, respectively, which are generated from the set $N_{f}$ by using a nonzero channelshifting seed. It can be observed that the first rendezvous occurs at the pivot channel, which is common to all the sequences. Therefore, TTR $=\frac{|N|}{2}$. In this scenario, ATTR and MTTR are same as TTR.

Theorem 1: MTTR between any two SUs in SymSyn CH protocol is $=\frac{|N|}{2}$.

Proof: In Lemma 2, it is proved that MTTR $=\frac{|N|}{4}$, when the first general sequence of two SUs is generated by using a zero and a nonzero channel-shifting seed. In Lemma 3, it is proved that MTTR $=\frac{|N|}{2}$, when the first general sequence of the two SUs is generated by using a nonzero channel-shifting seed. Hence, from Lemma 2 and Lemma 3, MTTR of SymSyn CH protocol can be calculated as $M A X\left(\frac{|N|}{4}, \frac{|N|}{2}\right)=\frac{|N|}{2}$.
3) Maximum Inter Rendezvous Interval (MIRI): From the channels structure of general sequence, it can be visualized that there is no rendezvous between the general sequences in their $N_{f}$ or $N_{b}$ part, if they are generated by using a nonzero channel-shifting seed applied on the set $N_{f}$ or $N_{b}$. If the order of the two consecutive general sequences for both SUs are generated from the channel-shifting seed other than 0 , applied

General Sequence $\mathrm{s}_{0}$ before applying shifting seed $k$


General Sequence $\mathrm{s}_{0}$ after applying shifting seed $k=2$
Fig. 5. Example of left circular Shift by $k=2$.
on set $N_{b}$ and $N_{f}$, respectively. In this scenario, rendezvous is only possible in the $N_{m}$ part of the second sequence. Hence, SUs need to wait for the entire length of the set $N_{b}$ for the first sequence + entire length of the set $N_{f}$ for the second sequence. Therefore, MIRI $=\left\lceil\frac{|N|}{2}\right\rceil+\left\lfloor\frac{|N|}{2}\right\rfloor \simeq|N|$. Considering an example for $|N|=9$, where $\left\langle s_{3}, s_{1}, s_{2}, s_{4}, s_{0}\right\rangle$ and $\left\langle s_{4}, s_{0}, s_{3}, s_{2}, s_{1}\right\rangle$ are the CH sequences of SU1 and SU2, respectively, MIRI $=$ $|N|-2$.

## B. SymAsyn CH Protocol

In asynchronous environment, SUs are assumed to enter into the network at different instant of time with knowledge about the number of available channels $|N|$ in the CRN. A timeslotted global clock system is considered, where duration of the global clock is equal to the length of the CH sequence. The local clock of each SU starts at the beginning of the global clock though it can enter into a channel at any instant of time represented in form of slot index. For the guaranteed rendezvous, the left circular shift (LCS) concept is introduced. The LCS operation for $k$ bits on a sequence of elements is defined as the circular shift of all elements towards left with a shifting of $k$ bits in clockwise. Consider a sequence $s$ having length $L_{s}$ such as $0,1, \ldots, k, \ldots, L_{s}-1$. After applying the LCS of $k$ bits on sequence $s$, the bits present in the sequence $s$ become $k, k+1, \ldots, L_{s}-1,0,1, k-1$. An example of LCS operation for $k=2$ on the general sequence $s_{0}$ is shown in Fig. 5. After applying LCS operation for $k=2$ bits, all channels of $s_{0}$ starting from channel 2 move by two positions towards left in clockwise, i.e., to channel 8. In SymAsyn CH protocol, each SU applies the LCS operation by $k$-bits in each general sequence and the value of $k$ is decided based on its entry slot index $\left(t_{e}\right)$. If $\left(\left\lfloor t_{e}\right\rfloor<|N|\right)$, $k=\left\lfloor t_{e}\right\rfloor$, else $k=\left(\left\lfloor t_{e}\right\rfloor \%|N|\right)$. The reason to introduce the concept of LCS is explained as below.

As shown in Fig. 6(a), there is no rendezvous between SU1 and SU2, if they enter into the network at different slot indexes. As shown in Fig. 6(b), LCS of 1-bit is applied on $s_{0}$ of SU1 and LCS of 2-bits is applied on the sequence $s_{4}$ of SU2. Here, the shifting parameter of 1-bit and 2-bits are nothing but the $t_{e}$ of SU1 and SU2. From Fig. 6(b), we can see that there must be some rendezvous of channels between the general sequences $s_{0}$ and $s_{4}$. The same mechanism can also be applied to the rest of the general sequences for both the SUs.

It is to be noted that initially the set of GS of each SU is generated by applying Algorithm 1. Then, each SU carries out the LCS operation based on its $k$ value, and updates the set of GS. Taking any permutation of those general sequences randomly,

Channel Hopping Sequence

(b)

Fig. 6. Rendezvous analysis in symmetric asynchronous scenario.

```
Algorithm 4: Generation of CH sequence in Symmetric
Asynchronous Environment.
Input: \(N\) : Set of available channels in the CRN.
    \(t_{e}\) : Stores entering slot index of a SU.
Output: CH_seq: Channel Hopping sequence.
```


## Notations:

```
\(G S\) : Set of general sequences; \(H S\) : Set that stores permuted order of general sequences; \(s\) : Set that stores individual general sequences before LCS; \(s_{l}\) : Set that stores individual general sequences after LCS; Seq: Set that stores the modified general sequences after LCS; \(k\) : Shifting seed; \(m\) : Stores the length of the GS.
    Initialize \(s=s_{l}=H S=G S=S e q=\)
    \(C H \_s e q=\Phi\);
    \(G S=\) general_sequence_generation \((N)\);
    \(m=|G S| ;\) sequm \(_{\text {num }}=0\);
    if \(\left\lfloor t_{e}\right\rfloor<|N|\) then
            \(k=\left\lfloor t_{e}\right\rfloor ;\)
    else
            \(k=\left\lfloor t_{e}\right\rfloor \%|N| ;\)
    end if
    while seq_num \(\leq m\) do
10: Process individual General Sequences from set GS
        which will store in set \(s\) before LCS operation;
        for \(i=0\) to \((|N|-1)\) do
            if \(k \leq(|N|-1)\) then
                \(s_{l}[i]=s[k] ; k=k+1 ;\)
            else
                    \(s_{l}[i]=s[n] ; n=n+1 ;\)
            end if
        end for
        Seq \(=\) Seq \(\| s_{l} ;\)
        \(s=s_{l}=\Phi\);
        seq_num \(=\) seq_num +1
    end while
    \(H S=H S \cup\{\) perm \((\) Seq \()\} ;\)
    \(C H \_\)seq \(=<C H \_s e q \| H S>\);
```

each SU generates its own CH sequence. The generation of CH sequence in SymAsyn environment is given in Algorithm 4.

Let $|N|=9$ be the number of available channels and $\mathrm{GS}=\left\{s_{0}, s_{1}, s_{2}, s_{3}, s_{4}\right\}$ be the set of general sequences generated by SU1 and SU2 by applying Algorithm 1. Hence, GS $=\left\{s_{0}:\langle 0,1,2,3,4,5,6,7,8\rangle, s_{1}:\langle 2,1,0,3,4,5\right.$,
$6,7,8\rangle, s_{2}:\langle 0,1,2,3,4,5,6,7,8\rangle, s_{3}:\langle 0,1,2,3,8,7,6,5,4\rangle$, $\left.s_{4}:\langle 0,1,2,3,6,5,4,8,7\rangle\right\}$. Let 3 and 20 be the $t_{e}$ of SU1 and SU2, respectively. Hence, the shifting seed of SU1 and SU2 is $k=3$ and $k=(20 \% 9)=2($ as $20>|N|)$, respectively. Now, each SU carries out the LCS operation on each general sequence based on its shifting seed $k$, and updates its GS. Now SU1 and SU2 create their own CH sequence by taking the random permutation of updated GS such as $<s_{2}, s_{1}, s_{4}, s_{3}, s_{0}>$ and $\left\langle s_{4}, s_{3}, s_{1}, s_{0}, s_{2}\right\rangle$, respectively. The possible rendezvous between SU1 and SU2 are shown in Fig. 7.

1) Degree of Rendezvous: In SymAsyn CH protocol, degree of rendezvous varies with the permutations of different GS and the clock_offsets of the SUs. In our protocol, there is guaranteed rendezvous between any two general sequences irrespective of the misalignment of the CH sequences. The degree of rendezvous can be calculated based on the following lemma.

Lemma 4: In SymAsyn CH protocol, maximum degree of rendezvous between any two SUs is $\{(m \times|N|)-2\}$, where $m$ is the number of general sequences generated from the $|N|$ number of available channels.

Proof: Let us consider an example to explain Lemma 4. Let number of available channels be $|N|=9$ and $m$ number of general sequences are generated from this, which can be given as GS $=\left\{s_{0}:\langle 0,1,2,3,4,5,6,7,8\rangle, s_{1}:\langle 2,1,0,3,4,5,6,7,8\rangle\right.$, $s_{2}:\langle 0,1,2,3,4,5,6,7,8\rangle, s_{3}:\langle 0,1,2,3,8,7,6,5,4\rangle, s_{4}:\langle 0,1,2$, $3,6,5,4,8,7\rangle\}$. By taking any pair of general sequences such as $\left\{s_{0}, s_{1}\right\}$ or $\left\{s_{2}, s_{3}\right\}$, there must be at least two nonoverlapping channels exist between them. If $t_{e}$ of SU1 and SU2 is 1 and 4, respectively, SU1 and SU2 will go for the LCS with $k=1$ and $k=4$, respectively. Let, $\left\langle s_{0}, s_{1}, s_{2}, s_{3}, s_{4}\right\rangle$ and $\left\langle s_{1}, s_{2}, s_{0}, s_{3}, s_{4}\right\rangle$ be the CH sequences of SU 1 and SU 2 , respectively, after taking the permutation. From the CH sequence of both SU1 and SU2, it is observed that rendezvous occurs in all slot indexes except the slot indexes where the pair of nonoverlapping channels are appeared such as $\{\langle 0\rangle,\langle 2\rangle\}$ and $\{\langle 2\rangle,\langle 0\rangle\}$. Hence, the degree of rendezvous in this case is $\{(5 \times 9)-2\}=43$, which can be generalized for any number of general sequences and available channels as $\{(m \times|N|)-2\}$.
2) Average Time To Rendezvous (ATTR) and Maximum Time To Rendezvous (MTTR): In an asynchronous environment, the calculation of TTR purely depends on the value of $t_{e}$ and the permuted set of GS. Therefore, we need to consider different cases considering $t_{e}$ value and the channels present in the set $N_{f}, N_{m}$, and $N_{b}$. Let us consider two SUs generate their first general sequence from the set $N_{f}$, one SU with channel-shifting seed $=0$ and other SU with a nonzero channel-shifting seed

## Channel Hopping Sequence



Fig. 7. Rendezvous between SUs where $t_{e}$ of SU1 is 3 i.e., $k=3$ and $t_{e}$ of SU2 is 20 i.e., $k=2$.
and let $t_{1}, t_{2}$ be the entry slot index $\left(t_{e}\right)$ of both the SUs, respectively.

Case 1: If $\left(0 \leq\left(t_{1}\right.\right.$ and $\left.\left.t_{2}\right) \leq \frac{|N|}{4}\right)$ and $\left|N_{f}\right|$ is odd:
Let us consider an example when the GS are generated for $|N|=16$ and $\left|N_{f}\right|=$ odd. Here, the set of general sequences $\mathrm{GS}=\left\{s_{0}:\langle 0, \mathbf{1}, \mathbf{2}, \mathbf{3}, 4,5,6,7,8,9,10,11,12,13,14,15\rangle\right.$, $s_{1}:\langle 6,5,4, \mathbf{3}, 2,1,0,7,8,9,10,11,12,13,14,15\rangle, s_{2}:\langle 4,3,2$, $1,0,6,5,7,8,9,10,11,12,13,14,15\rangle, s_{3}:\langle 2,1,0,6,5,4,3$, $7,8,9,10,11,12,13,14,15\rangle, s_{4}:\langle 0,1,2,3,4,5,6,7,8,9,10$, $\mathbf{1 1}, 12, \mathbf{1 3}, \mathbf{1 4}, \mathbf{1 5}\rangle, s_{5}:\langle 0,1,2,3,4,5,6,7,14,13,12,11,10$, $9,8, \mathbf{1 5}\rangle, s_{6}:\langle 0,1,2,3,4,5,6,7,12,11,10,9,8,15,14,13\rangle$, $\left.s_{7}:\{0,1,2,3,4,5,6,7,10,9,8,15,14,13,12,11\rangle\right\}$. The channels present in the set $N_{f}, N_{b}$, and $N_{m}$ are $\{0,1,2,3,4,5,6\}$, $\{8,9,10,11,12,13,14,15\}$, and $\{7\}$, respectively. In the set GS, $s_{0}$ is generated from the set $N_{f}$ by using the channelshifting seed $=0$, whereas $s_{1}, s_{2}$, and $s_{3}$ are generated from $N_{f}$ by applying the nonzero channel-shifting seeds. If combination of sequences such as $\left\{s_{0}, s_{1}\right\}$ or $\left\{s_{0}, s_{2}\right\}$ or $\left\{s_{0}, s_{3}\right\}$ is taken, then there must be some overlapping of channels. Hence, $\operatorname{TTR}=1,2, \ldots, \frac{|N|}{4} . \mathrm{ATTR}=\frac{\frac{|N|}{4}+1}{2}$ and MTTR $=$ MAX (all TTRs between SUs) $=\frac{|N|}{4}$.

Case 2: If $\left(\frac{|N|}{4}<\left(t_{1}\right.\right.$ and $\left.\left.t_{2}\right)<\frac{|N|}{2}\right)$ and $\left|N_{f}\right|$ is odd:
Here, TTR $=2, \ldots, \frac{|N|}{4}$, as there is no rendezvous in the $N_{f}$ part of the sequence. In this case, rendezvous occurs at the channel present in the set $N_{m}$, i.e., at the channel number 7, as $N_{m}$ is $\{7\}$. Hence, $\operatorname{ATTR}=\frac{\frac{|N|}{4}+2}{2}$ and MTTR $=$ MAX (all TTRs between SUs $)=\frac{|N|}{4}$. The same scenarios can also be analyzed for $\left|N_{f}\right|=$ even as follows.

Case 3: If $\left(0 \leq\left(t_{1}\right.\right.$ and $\left.\left.t_{2}\right) \leq \frac{|N|}{4}\right)$ and $\left|N_{f}\right|$ is even:
Let us consider the GS generated from $|N|=18$, where $\left|N_{f}\right|=$ even, i.e., $\left|N_{f}\right|=\{0,1,2,3,4,5,6,7\}$. Here, the set of general sequences $G S=\left\{s_{0}:\langle 0, \mathbf{1}, \mathbf{2}, \mathbf{3}, 4, \mathbf{5}, \mathbf{6}, 7,8,9,10,11\right.$, $12,13,14,15,16,17\rangle, s_{1}:\langle 6,5,4,3,2,1,0,7,8,9,10,11,12$, $13,14,15,16,17\rangle, s_{2}:\langle 4,3,2,1,0,7,6,5,8,9,10,11,12,13$, $14,15,16,17\rangle, s_{3}:\langle 2,1,0,7,6,5,4,3,8,9,10,11,12,13$, $14,15,16,17\rangle, s_{4}:\langle 0,1,2,3,4,5,6,7,8,9,10,11,12,13$, $14,15,16,17\rangle, s_{5}:\langle 0,1,2,3,4,5,6,7,8,17,16,15,14,13$, $12,11,10,9\rangle, s_{6}:\langle 0,1,2,3,4,5,6,7,8,15,14,13,12,11$, $10,9,17,16\rangle, s_{7}:\langle 0,1,2,3,4,5,6,7,8,13,12,11,10,9,17$, $16,15,14\rangle, s_{8}:\langle 0,1,2,3,4,5,6,7,8,11,10,9,17,16,15$, $14,13,12\rangle\}$. The channels present in the set $N_{f}, N_{b}$, and $N_{m}$ are $\{0,1,2,3,4,5,6,7\},\{9,10,11,12,13,14,15,16,17\}$, and $\{8\}$, respectively.

In this scenario, the values of TTRs are same with the case of $\left|N_{f}\right|$ is odd. Hence, $\operatorname{TTR}=1,2,3, \ldots, \frac{|N|}{4}, \mathrm{ATTR}=\frac{\frac{|N|}{4}+1}{2}$ and $\operatorname{MTTR}=\frac{|N|}{4}$.

Case 4: If $\left(\frac{|N|}{4}<\left(t_{1}\right.\right.$ and $\left.\left.t_{2}\right)<\frac{|N|}{2}\right)$ and $\left|N_{f}\right|$ is even.
If different combinations of general sequences are taken, then TTR $=1,2, \ldots, \frac{|N|}{4}$. Hence, ATTR $=\frac{\frac{|N|}{4}+1}{2}$ and MTTR $=\frac{|N|}{4}$.

However, if both the sequences of the set $N_{f}$ are generated by using the nonzero channel-shifting seeds and the values of $t_{1}$ and $t_{2}$ are $0 \leq\left(t_{1}\right.$ and $\left.t_{2}\right)<\frac{|N|}{2}$, there is no rendezvous in the $N_{f}$ part of the general sequence, and rendezvous will occur among the channels present in the set $N_{m}$. Hence, $\operatorname{TTR}=2,3, \ldots, \frac{|N|}{2}$. Therefore, ATTR $=\frac{\frac{|N|}{2}+2}{2}$ and MTTR $=\frac{|N|}{2}$. If initial general sequences of both SUs are generated from the set $N_{b}$ and $0 \leq$ $\left(t_{1}\right.$ and $\left.t_{2}\right)<\frac{|N|}{2}$, then TTR $=1$, as the channels present in the set $N_{f}$ are same for all the general sequences generated from the set $N_{b}$. The same analysis of TTR, ATTR, and MTTR can be done for $\frac{|N|}{2}<\left(t_{1}\right.$ and $\left.t_{2}\right) \leq(|N|-1)$.

Theorem 2: MTTR in SymAsyn CH protocol is $\approx \frac{3 \times|N|}{4}$.
Proof: Consider SU1 and SU2, which are associated with clock_off sets $t_{1}$ and $t_{2}$, respectively, where $t_{2}>t_{1}$. The worst case of TTR arises, if $t_{1}=0$ and $t_{2}=\left(\frac{|N|}{2}+1\right)$ or due to the permuted order of general sequences. Let SU1 be at the general sequence $s_{i}$ when SU2 enters to the network with initial general sequence $s_{j}$. If both sequences $s_{i}$ and $s_{j}$ are generated from the set $N_{b}$ after applying the nonzero channel-shifting seeds, then no rendezvous occurs in the $N_{b}$ part of both the sequences. Rendezvous occurs in the $N_{f}$ part of next general sequence for SU1 with the $N_{f}$ part of the sequence $s_{j}$ for SU2. Hence, both SUs need to wait for the entire $N_{b}$ part and maximum $\frac{|N|}{4}$ number of slot indexes in the $N_{f}$ part. Therefore, MTTR can be calculated as $\left(\left\lceil\frac{|N|}{2}\right\rceil\right)+\left(\frac{|N|}{4}\right) \approx \frac{3 \times|N|}{4}$.
3) Maximum Inter Rendezvous Interval (MIRI): In SymAsyn CH protocol, the MIRI between any two SUs is $\approx \frac{3 \times|N|}{4}$, which is obtained from the analysis of MTTR.

## V. Performance Evaluation

Performance of our protocols is evaluated by using OMNeT++ simulator IDE version 4.5 and is compared with some related works such as RCCH [1], ARCH [1], CACH [12], jump stay (JS) [13], L-QCH [14], FRCH [17], EJS [19] published in well-known journals. In the simulation, 50 PUs and 100 SUs are assumed to be deployed over an area of


Fig. 8. Throughput versus number of channels.
$1000 \mathrm{~m} \times 1000 \mathrm{~m}$. The entire network is divided into multiple number of time slots, where duration of each time slot is taken as 0.02 s . The study of the impact of number of channels on throughput, ATTR and \% of rendezvous in the synchronous and asynchronous environment is carried out. The throughput is defined as the number of packets successfully transmitted between the sender and receiver per unit time.

## A. Evaluation of Symmetric Synchronous Protocol

In Fig. 8, the impact of number of channels on the throughput is evaluated. In our SymSyn CH Protocol, there must be at least $\frac{N}{2}$ number of rendezvous between any two general sequences generated either from the set $N_{f}$ or $N_{b}$. Besides, the degree of rendezvous between two SUs is $(m *|N|)$, i.e., $100 \%$ when they choose the same set of permuted general sequences, where $m$ is the number of general sequences. Hence, the degree of rendezvous in SymSyn protocol increases with increase in the number of available channels, which gives more opportunity to the SUs for data communication. In RCCH, the throughput decreases gradually as the probability of rendezvous is very less, i.e., $\frac{2}{N}$. The throughput decreases exponentially, since only one channel acts as the rendezvous one, which may lead to the channel saturation problem. In L-QCH, the throughput decreases with the number of channels as only one rendezvous occurs per frame. As we know, JS protocol, contains one stay and one jump pattern. In jump pattern, rendezvous probability is very low and there is no rendezvous in the stay pattern, if both SUs can have different stay channels. Hence, the throughput of JS is very small as compared to other protocols.

The impact of number of SUs on the throughput is evaluated, as shown in Fig. 9. Throughput of L-QCH and CACH is very small due to the channel congestion problem as rendezvous occur only in a particular channel for the entire cycle or a frame. In RCCH, throughput decreases due to low probability of the rendezvous and very limited number of availability of alternate sequences, which increases the number of collision. In JS, the throughput is very less as the number of rendezvous is not uniform throughout the sequence. In SymSyn, the throughput in multiuser scenario is very high due to less chance of collision among SUs. In SymSyn, it is unlikely that two or more SUs can have same permutations of general sequences as plenty of permuted general sequences are available. Besides, each


Fig. 9. Throughput versus number of secondary users.


Fig. 10. \% of rendezvous versus number of channels.
permutation can guarantee at least $\frac{N}{2}$ number of rendezvous between any two general sequences generated either from the set $N_{f}$ or $N_{b}$. Due to constant number of channels considered in our simulation, the throughput decreases with increase in the number of SUs. However, our protocol outperforms in comparison to others.

As shown in Fig. 10, it is observed that $\%$ of rendezvous in our protocol is very high as there must be at least $\frac{N}{2}$ number of rendezvous between any two general sequences generated from different channel-shifting seeds either from the set $N_{f}$ or $N_{b}$. Besides, maximum degree of rendezvous in SymSyn protocol is $\{(m-2) *|N|\}+\left\{\left(2 \times\left(\left\lceil\frac{|N|}{2}\right\rceil+1\right)\right)\right\}$, which is much higher than other protocols, where $m$ is the number of general sequences. Moreover, the degree of rendezvous between two SUs can be $100 \%$, when they choose the same sets of permuted general sequences. In RCCH, percentage of rendezvous decreases exponentially with the increase in the number of channels as the probability of rendezvous is only $\frac{2}{N}$. In L-QCH, there is no increment in the number of rendezvous though length of the frame is increased with channel numbers. In JS, its very difficult to find a common available channel for the rendezvous due to its blind rendezvous nature. In CACH , the probability of rendezvous is very less, i.e., $N$ out of $N \times(N+1)$ time slots.

The impact of number of channels on ATTR is shown in Fig. 11. ATTR of SymSyn in most of the scenarios is $\leq \frac{N}{8}$ with guaranteed rendezvous within $\frac{N}{2}$ numbers of slots. Hence, SUs need not to wait for more numbers of time slots


Fig. 11. Average time To rendezvous (ATTR) versus number of channels.


Fig. 12. Throughput versus number of channels.
for the rendezvous. In RCCH , the length of one cycle is $N$ and number of rendezvous per cycle is 2 . Hence, a SU has to wait for more number of time slots for its rendezvous with increase in the value of $N$, which increases the value of ATTR. In L-QCH, the frame length increases with increase in the channel number. The probability of rendezvous is very low per frame, which increases the value of ATTR. In CACH, value of ATTR increases as there is only one rendezvous within $N+1$ intervals. In JS, it is very difficult to find one common channel within $3 P$ time slots, which includes both jump and stay patterns. Therefore, the value of ATTR is very high as compared to other protocols.

## B. Evaluation of SymAsyn Protocol

As shown in Fig. 12, it is clear that the throughput of ARCH, JS, and EJS is very low as compared to our SymAsyn due to less degree of rendezvous and large value of MTTR. In FRCH, the degree of rendezvous is very small with length of the CH sequence for which the number of transmissions between any two SUs is decreased. In our SymAsyn protocol, throughput is very high due to higher degree of rendezvous, i.e., the average degree of rendezvous is more than $50 \%$, though it is not uniform due to permutation type and $t_{e}$ of the SUs.

The impact of number of channels on $\%$ of rendezvous is presented in Fig. 13. In ARCH , the number of rendezvous is very small, i.e., only $N$ out of $N^{2}$ time slots, which decreases the number of rendezvous with increase in channel numbers.


Fig. 13. \% of rendezvous versus number of channels.


Fig. 14. Average time To rendezvous (ATTR) versus number of channels.

Degree of rendezvous in JS and EJS is not uniform as there is no guaranteed rendezvous within a bounded interval, which decreases the percentage of rendezvous. In FRCH, degree of rendezvous is very small as compared to the length of the CH sequence. Besides, control channel saturation problem arises, which degrades the performance. However, the degree of rendezvous in SymAsyn protocol increases with increase in the number of channels as well as is greater than the other protocols. The maximum degree of rendezvous of SymAsyn protocol is $\{(m \times|N|)-2\}$, whereas the average degree of rendezvous is $>50 \%$.

The impact of number of channels on ATTR is evaluated, as shown in Fig. 14. In JS and EJS, the value of TTR is $\approx 3 P$ and $\approx 4 P$, respectively, by which the value of ATTR is increased. In ARCH and FRCH, value of ATTR is very large as the probability of the rendezvous is very low, i.e., $\frac{1}{N}$ and $\frac{N}{2 \times(N+1)}$, respectively. Therefore, SUs have to wait more number of time slots. The value of ATTR in SymAsyn varies with the entry slot index of the SUs. However, the value is very small in most of the scenarios, i.e., $\leq \frac{N}{4}$.

## C. Comparison Between SymSyn and SymAsyn Protocol

In order to compare the performance of our protocols SymSyn and SymAsyn, simulations are performed for the degree of rendezvous and ATTR, as shown in Figs. 15 and 16,


Fig. 15. Comparison of SymSyn and SymAsyn protocols in terms of \% of rendezvous.


Fig. 16. Comparison of SymSyn and SymAsyn protocols in terms of ATTR.
respectively. The degree of rendezvous of both of our protocols is almost same and the values lies between $70 \%$ and $100 \%$. As shown in Fig. 15, it can be observed that SymSyn shows better performance as the degree of rendezvous in SymSyn depends on the permutation of general sequences. In SymSyn, if two SUs choose the same set of general sequences simultaneously, the chance of degree of rendezvous is more. However, it varies with the entry slot index $\left(t_{e}\right)$ of SUs in SymAsyn. As shown in Fig. 16, the value of ATTR for both of our protocols is very small as compared to other protocols. However, SymSyn performs little better in terms of ATTR, i.e., $\leq \frac{N}{2}$ than SymAsyn as ATTR varies with entry slot index of both SUs and lies within $0 \leq \frac{3 \times N}{4}$ in SymAsyn.

## VI. Conclusion

In this paper, two novel CH mechanisms are designed to optimize the degree of rendezvous, MTTR, ATTR, and MIRI in ICRN. Theoretical analysis of our proposed SymSyn and SymAsyn CH protocols are made to justify the correctness and to measure their performance. From the performance evaluation, it is observed that our protocols can achieve highest degree of rendezvous with minimum MTTR and MIRI. As compared to similar related works, our algorithms outperforms over other
related protocols for both symmetric synchronous and asynchronous environment. Hence, our proposed protocols can be considered as the optimal solution for achieving higher degree of rendezvous with smaller value of MTTR and ATTR, which is highly essential for establishing the common control channel between two SUs in ICRN.

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