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# TRANSMISSION OF THERMAL ENERGY IN MAGNETO-HYDRODYNAMIC UNSTEADY FREE CONNECTIVE FLOW OF MERCURY AND LIQUID SODIUM PAST AN INFINITE POROUS FLAT PLATE IN PRESENCE OF HEAT ABSORBING SINKS WITH CONSTANT SUCTION

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*The computational analysis of heat transfer in the unsteady free convective flow of mercury and liquid sodium past an infinite vertical plate with constant suction and heat absorbing sinks has been carried out in presence of an external transverse magnetic field. Approximate solutions have been derived for the velocity and temperature fields, mean skin-friction and mean rate of heat transfer. It is observed that increase in magnetic field strength decreases the mean skin-friction and mean rate of heat transfer both for mercury and liquid sodium.*

**KEY WORDS :** MHD, thermal energy, unsteady, porous flat, Plate, Sinks, constant suction.

**NOMENCLATURE :**

$g$ = acceleration due to gravity

$(X', Y')$ = Cartesian coordinates

$(u', V')$ = Velocity components of the fluid along $X'$ and $Y'$ axes

$V_0'$ = Velocity of suction

$T'$ = temperature of the fluid

$T_m'$ = ambient fluid temperature.

$T_\omega'$ = temperature of the plate

$K$ = thermal diffusivity

$\beta$ = co-efficient of volume expansion

$\nu$ = kinematic viscosity

$\sigma$ = electrical conductivity

$C_v$ = specific heat of the fluid of constant pressure

$\rho$ = density of the fluid

$\omega'$ = angular frrequency of oscillation

$\sqrt{-1} = i$ = the imaginary quantity

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$P$ = Prandtl number

$G$ = Grashof number

$S$ = Sink parameter

$E$ = Eckert number

$M$ = Square of Hartmann number

**INTRODUCTION :** The unsteady free convection flow past an infinite plate with constant suction and heat sources has been studied by Pop and Soundalgekar [1]. The effect of magnetic field on free convective flow of electrically conducting fluids past on a semi-infinite flat plate has been analysed by Gupta [2], Singh and Cowling [3], Nanda and Mohanty [4]. Sacheti, Chandran and Singh [5] have obtained an exact solution for the unsteady MHD problem. But they have neither considered the effects of constant suction nor included the heat absorbing sink and viscous dissipation. But the propagation of thermal energy through mercury and liquid sodium in presence of external magnetic field and heat absorbing sinks has wide range of applications in chemical and aeronautical engineering, atomic propulsion space science etc. Hence our objective in the present paper is to study the heat transfer in mercury ($P = 0.025$) and liquid sodium ($P \doteq 1.0$) past an infinite porous plate with constant suction including viscous dissipation in the presence of uniform transverse magnetic field and heat sinks.

**MATHEMATICAL ANALYSIS :** Let the $x'$-axis be taken in the vertically upward direction along the infinite vertical plate and $y'$-axis normal to it. Neglecting the induced magnetic field and applying Boussinesq's approximation, the equations of the flow can be written as :

Continuity : $$\frac{\partial V'}{\partial y'} = 0 \qquad \ldots (1)$$

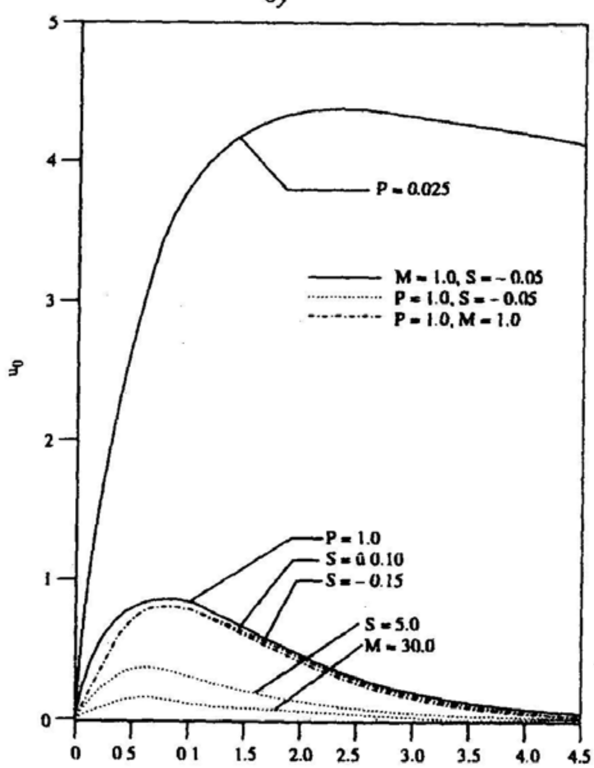


**Fig. 1. Effects of $P$, $M$ and $S$ on mean velocity for $G = 5.0$, $E = 0.001$, $w = 5.0$, $\varepsilon = 0.2$, $wt = \pi/2$.**

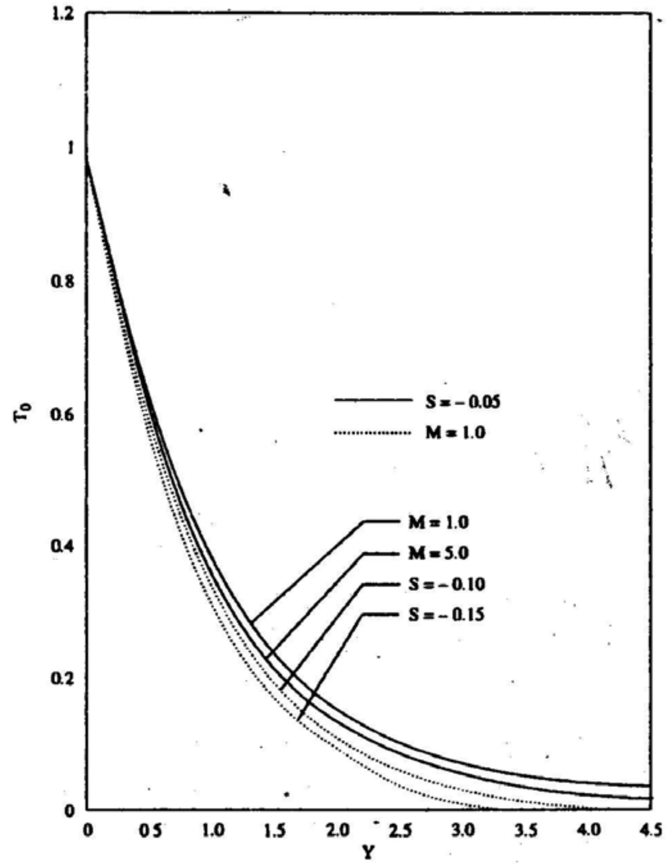


**Fig. 2. Effects of $M$ and $S$ on mean temperature for $G = 5.0$, $P = 1.0$, $E = 0.001$, $w = 5.0$, $\varepsilon = 0.2$, $wt = \pi/2$.**

*i.e.*, $$V' = V_0' \text{ (constant)} \qquad \dots (2)$$

Momentum : $$\frac{\partial V'}{\partial t'} + V' \frac{\partial u'}{\partial y'} = g\beta (T' - T_\infty') + \frac{\nu \partial^2 u'}{\partial y'^2} - \frac{\sigma B_0^2}{\rho} u \qquad \dots (3)$$

Energy : $$\frac{\partial T'}{\partial t'} + V' \frac{\partial T'}{\partial y'} = \frac{\partial^2 T'}{\partial y'^2} + S' (T' - T_\infty') + \frac{\nu}{C_p} \left( \frac{\partial u'}{\partial y'} \right)^2 \qquad \dots (4)$$

Since the externally applied magnetic field strength is weak, the Joulean heat dissipation is disregarded. The boundary conditions of the problem are

$$u' = 0,\ V' = - V_0',\ T' = T_\omega' + \varepsilon (T_\omega' - T_\infty') e^{i\omega' t'} \quad \text{at} \quad y' = 0 \qquad \dots (5)$$

$$u' \to 0,\ T' \to T_\infty' \quad \text{as} \quad y' \to \infty$$

**METHOD OF SOLUTION :** Introducing the following non-dimensional parameters

$$y = \frac{y' V_0'}{\nu},\ t = \frac{t' V_0'^2}{4\nu},\ \omega = \frac{4\nu\omega'}{V_0'^2},\ u = \frac{u'}{V_0'}$$

$$T = \left( \frac{T' - T_\infty'}{T_\omega - T_\infty'} \right),\ P = \frac{\nu}{K},\ G = \frac{\nu g \beta (T_\omega' - T_\infty')}{V_0'^2} \qquad \dots (6)$$

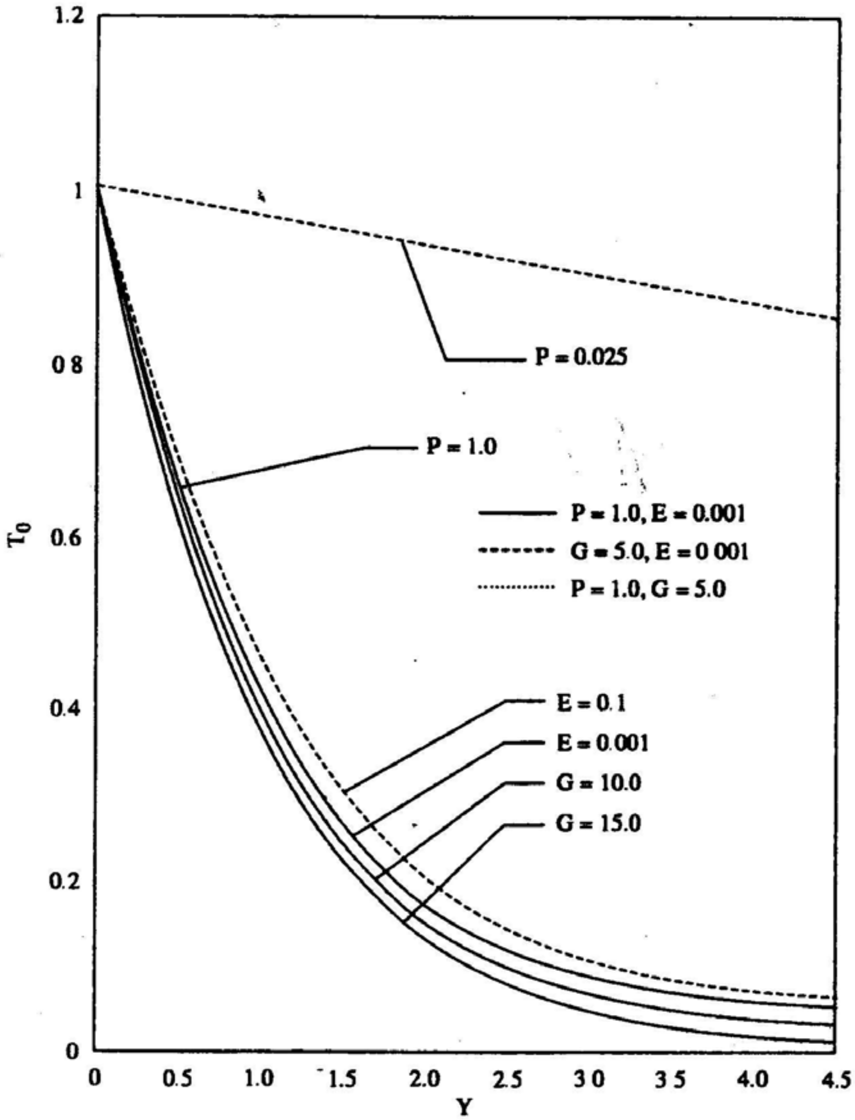


**Fig. 3. Effect of *P*, *G* and *E* on mean temperature for $M = 1.0$, $S = -0.05$, $w = 5.0$, $\varepsilon = 0.2$, $wt = \pi/2$.**

$$S = \frac{4S'\nu}{V_0'^2},\quad E = \frac{V_0'^2}{C_p\,(T_\omega' - T_\infty')},\quad M = \left(\frac{\sigma B_0^2}{\rho}\right)\frac{\nu}{V_\infty'^2}$$

and with the help of (5) and (6), equations (2) and (3) become

$$\frac{1}{4}\cdot\frac{\partial u}{\partial t} - \frac{\partial u}{\partial y} = GT + \frac{\partial^2 u}{\partial y^2} - Mu \qquad \ldots (7)$$

$$\frac{P}{4}\cdot\frac{\partial T}{\partial t} - p\,\frac{\partial T}{\partial y} = \frac{\partial^2 T}{\partial y^2} + \frac{PS}{4}T + PE\left(\frac{\partial u}{\partial y}\right)^2 \qquad \ldots (8)$$

and the modified boundary conditions are

$$u = 0,\; T = 1 + \varepsilon e^{i\omega t} \quad \text{at} \quad y = 0 \qquad \ldots (9)$$

$$u \to 0,\; T \to 0 \quad \text{as} \quad y \to \infty$$

To solve equations (7) and (8), we assume the velocity and temperature in the neighbourhood of the plate as

$$u(y, t) = u_0(y) + \varepsilon e^{i\omega t} u_1(y);$$

$$Y(\chi, t) = T_0(y) + \varepsilon e^{i\omega t} T_1(y) \qquad \ldots (10)$$

Putting (10) in equations (7) and (8), equating harmonic and non-harmonic terms and neglecting the co-efficient of $\varepsilon^2$, we get

$$u_0'' + u_0' - M_0 u_0 = -GT_0,\ T_0'' + PT_0' + \frac{PS}{4} T_0 = -PE\left(\frac{\partial u_0}{\partial y}\right)^2 \qquad \ldots (11)$$

and

$$u_1'' + u_1' - \frac{i\omega}{4} u_1 - Mu_1 = -GT_1,$$

$$T_1'' + PT_1' - \frac{P}{4}(i\omega - S) T_1 = -2PE\left(\frac{\partial u_0}{\partial y}\right)\left(\frac{\partial u_1}{\partial y}\right) \qquad \ldots (12)$$

Now the corresponding boundary conditions are

$$y = 0 : u_0 = 0 = u_1,\ \ T_0 = 1 = T_1$$

$$y \to \infty : u_0 = 0 = u_1,\ T_0 = 0 = T_1 \qquad \ldots (13)$$

Using multiparameter perturbation technique, assuming $E << 1$, the modified boundary conditions, we can express the velocity and temperature fields in terms of the fluctuating parts as follows :

$$u = u_0 + \varepsilon(M_r \cos \omega t - M_1 \sin \omega t),$$

$$T = T_0 + \varepsilon(T_1 \cos \omega t - T_1 \sin \omega t) \qquad \ldots (14)$$

where

$$u_1 = M_r + iM_i,\ T_1 = T_r + iT_i \qquad \ldots (15)$$

The transient velocity and temperature for $\omega t = \pi/2$ are given by

$$u = u_0 - \varepsilon M \quad \text{and} \quad T = T_0 - \varepsilon T \qquad \ldots (16)$$

The skin friction at the plate in dimensionless form is given by

$$\tau_\omega = \left(\frac{\partial u}{\partial y}\right)_{y=0} = u_0'(0) + \varepsilon e^{i\omega t} u_1'(0) \qquad \ldots (17)$$

Separating real and imaginary parts of equation (17) and taking real part only, we have

$$\tau_\omega = \tau_\omega^m + \varepsilon\,|B|\cos(\omega t + \alpha), \qquad \ldots (18)$$

where

$$|B| = \sqrt{B_r^2 + B_i^2},\ \alpha = \tan^{-1}\left(\frac{B_i}{Br}\right) \qquad \ldots (19)$$

Similarly, the rate of heat transfer at the plate is given by

$$q_\omega = \left(\frac{\partial T}{\partial y}\right)_{y=0} = T_0'(0) + \varepsilon\, e^{i\omega t} T_1'(0) \qquad \ldots (20)$$

and on further simplification we have

$$q_\omega = q_\omega^m + \varepsilon\,|H|\cos(\omega t + \gamma) \qquad \ldots (21)$$

where $$|H| = \sqrt{H_r^2 + H_i^2}, \quad \gamma = \tan^{-1}\left(\frac{H_i}{H_r}\right) \qquad \dots (22)$$

**RESULTS AND DISCUSSIONS :** The mean velocity and mean temperature profiles are shown in figures 1, 2 and 3. It is observed that increase in magnetic field strength decreases the mean velocity and mean temperature of mercury ($P = 0.025$) and liquid sodium ($P = 1.0$). Also increase in sink strength leads to decrease the mean velocity and temperature for liquid sodium. From figure 3 it is analysed that the temperature falls if liquid sodium is taken in place of mercury keeping elastic parameter $E$, magnetic parameter $M$ and sink strength $S$, constant. But the temperature of the liquid sodium increases with increase in elastic parameter ($E$).

It is noticed from the table–1 that the increase in magnetic field strength and sink strength decreases mean skin friction both for mercury and liquid sodium. Also the same effect is observed in case of mean rate of heat transfer both for the conducting fluids.

**Table–1 : Values of mean skin friction ($\tau_\omega^m$) and mean rate of heat transfer ($q_\omega^m$) for $G = 5.0, E = 0.001, \omega = 5.0, \varepsilon = 0.2$, wt. $\pi/2$.**

| P | M | S | $\tau_\omega^m$ | $q_\omega^m$ |
|---|---|---|---|---|
| Mercury ($P = 0.025$) | 1.0 | – 0.05 | 7.6674 | – 0.0336 |
| | 5.0 | – 0.05 | 2.7391 | – 0.0341 |
| | 5.0 | – 0.10 | 2.7297 | – 0.0404 |
| Liquid sodium ($P = 1.0$) | 1.0 | – 0.05 | 3.0682 | – 0.9330 |
| | 5.0 | – 0.05 | 1.7835 | – 1.0089 |
| | 5.0 | – 0.10 | 1.7759 | – 1.0209 |

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